Global Economy and Finance Journal
Vol. 3 No. 1 March2010, Pp. 1-13

Stock Selection by Means of DEA and Stochastic Dominance

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This paper investigates the problem of statistical testing for second order stochastic dominance (SSD) relations among a set of stocks. The SSD rule is the desirable condition that all risk averse, non-satiated investors seek. Due to the fact that SSD is not significantly confirmed by statistical tests, in most cases, weak dominance of some of stocks over others cannot be rejected at a high level of confidence (95% or higher). For this reason, we have tested the necessary rules of SSD efficiency incorporated into data envelopment analysis (DEA) models, recently proposed, on SSD statistical test results to remove the stocks that are weak SSD efficient. Weak SSD efficiency refers to stocks that are never dominated by others and reported as SSD efficient, but they are dominated by a combination of other stocks given the necessary rules of SSD efficiency. Based on an empirical study of 68 financial stocks from the S&P 500 Index, six stocks were determined to be SSD efficient. Applying DEA, this number was reduced to three. These three stocks are desirable choices for the class of investors who prefer having more wealth to less and are risk averse.

Field of Research: Security Analysis, Stock Selection

1. Introduction

In today's fluctuating market, stock selection is crucial to evaluate the profitability of an investment and to limit the financial risk of the investment. Moreover, realistic selection criteria will help fund managers to pick, and heavily weight, the stocks with superior performance and thus build a strong portfolio. In this context, the accuracy of the selection method seems to be as important as the performance of the stock itself. Therefore, several selecting methods have been proposed. Markowitz (1959) proposed the mean-variance analysis (MV), which, although proven to be useful, only considers the mean and variance and neglects other important factors such as skewness and kurtosis of stock return distribution. An alternative for the

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mean-variance analysis is second order stochastic dominance (SSD). This approach is based on relations between pairs of the cumulative distribution functions (CDF) of stock returns. The SSD rule is the desirable condition that all risk averse, non-satiated investors seek. Since the SSD is not vigorously confirmed by statistical tests, often weak dominance of some stocks over others cannot be rejected at a high level of confidence (95% or higher). The work presented here proposes to address the issue of SSD statistical testing by removing the weak SSD efficient stocks. This paper is organized as follows. In Section 2 we present a brief review of stock selection methods. Next, we introduce SSD, DEA and DEA-SSD models. In section 4, we apply the SSD and DEA test to the daily returns of a number of financial stocks obtained from the S&P 500 Index. We explain the methodology which we use in the empirical study and analyse the test results. Conclusions are provided in section 5.

2. Literature Review

Expected utility maximization is the main approach used for making decision on investment options such as stocks (Tversky and Kahneman, 1992; Sahalia and Brandt, 2000). In this approach, an investor needs to define his/her preference (utility function) to differentiate the best stock among an admissible set of choices. If full information of investors' preferences is available, expected utility analysis will result in the complete ordering of stocks under consideration with respect to their utility value and the stock with the highest expected utility will be chosen. However, most of the time, only partial information on investors' preferences is available. Therefore, one can look for an approach that can employ this partial information to partially differentiate stocks. A number of methods have been developed; each has its pros and cons. The Mean-Variance (MV) approach of Markowitz (1952) proved to be a significant contribution, but has certain limitations. Specifically, it only considers the mean and variance of the stock return distributions (or assumes distributions are normal) and neglects other important distribution parameters such as skewness and kurtosis.

Second order stochastic dominance (SSD), (Hadar, 1969; Hanoch, 1970) is a more universal rule that is often referred to as the basic concept of decision theory. The main advantage of this approach, compared to the MV analysis, is that the SSD makes no assumptions regarding the return distributions of stocks. However, it needs the full information of stock distributions, which often cannot be obtained empirically. In the actual application of the method, the distributions of stocks have to be inferred from historical observations of the returns to form empirical distribution functions. Due to the high frequency of false rejection of SSD by empirical distributions, several statistical tests have been developed for surveying the SSD relationship (Beach & Davidson, 1983; Deshpande & Singh, 1985; Chow, 1989; Zheng et al., 2000). In many tests, it is assumed that there is no dependence within and between the samples, which is not true in many applications, particularly with time series data such as financial data. For this reason, a number of other statistical tests (Schmid and Trede, 1997; Fisher and Willson, 1997; Linton et al., 2005;

Klaver, 2006) were developed. These tests are applicable for data with possible dependence structures within and between their samples. The majority of tests consider the null hypothesis of SSD dominance and the alternative of non-SSD dominance. However, in these tests, SSD is not confirmed with statistical significance and, in most cases, weak dominance of some stocks over others cannot be rejected with a high level of confidence (95% or higher). Here we aim to solve this problem by applying a DEA analysis on the results obtained from a SSD test.

The DEA models presented here are built on the work of Lozano and Gutierrez (2007). Lozano et al. proposed incorporating the SSD necessary rules into the DEA approach to find stocks that are candidates for SSD efficiency. However, our methodology applies DEA models proposed by the aforementioned authors on the stocks that are known to be SSD efficient and thus removes the weakly SSD efficient stocks. Weak SSD efficiency refers to stocks that are never dominated by others and reported as SSD efficient, but they are dominated by a combination of other stocks.

3. Methodology and Research Design

3.1 Stochastic Dominance

Stochastic dominance (SD) is a basic concept of decision theory. This approach is based on the relations between pairs of cumulative distribution functions (CDF) of stock returns. A SD efficient stock is not dominated by other stocks and a SD inefficient stock is at least dominated by one other stock. Three different orders of SD have been developed based on three different utilities, or, equivalently, three different types of investors. The First Order SD (FSD) rule (Quirk & Saposnik, 1962; Fishburn, 1964; Hadar, 1969; Hancoch & Levy, 1970) is for those investors who always prefer having more wealth to less. Therefore, either a conservative investor or a gambler prefers a FSD efficient stock over a FSD inefficient one.

Mathematically, FSD can be explained as follows. Assume that the CDFs of returns for stocks A and B are denoted by Φ_A and Φ_B, respectively. The CDF of A will be preferred over the CDF of B by FSD efficiency if and only if, for every return value, R, $\Phi_B(R) \geq \Phi_A(R)$ In other words, the probability of having a return less than R is higher for stock B than for stock A. In general, the relationship between distributions for higher orders of SD can be defined as follows,

$$D^1 = \Phi(R) \ ,$$

$$D^2(R) = \int_0^R D^1(z)dz \ , \qquad (1)$$

$$D^{s+1}(R) = \int_0^R D^s(z)dz \qquad for \ \ s = 1,2,3,... \ .$$

The CDF of returns of stock A dominates that of stock B for a SD of the order S if and only if $D^s_B(R) \geq D^s_A(R)$ for all values of R. Based on this definition, stock A will be preferred to B by Second Order SD (SSD) if and only if $D^2_B(R) \geq D^2_A(R)$ and will be preferred over B by Third Order SD (TSD) if and only if $D^3_B(R) \geq D^3_A(R)$. The SSD rule (Hadar, 1969; Hanoch, 1970) refers to investors who prefer more wealth to less and are risk averse. The TSD rule (Whitmore, 1970) refers to investors who prefer more wealth to less, are risk averse, and have declining risk aversion with increasing wealth.

3.2 Data envelopment analysis based on second order stochastic dominance

Data envelopment analysis (DEA) has been found to be very useful for evaluating the relative performance of a given set of stocks. This approach can assess the relative performance of stocks in terms of efficiency scores (Murthi et al., 1997; Choi & Murthi, 2001; Gregoriou et al., 2005). Different authors have used different approaches for selecting DEA inputs and outputs. Most of the approaches used a measure of risk as an input and the mean return as an output. In order to test the SSD relationship, Lozano et al. proposed different linear DEA models based on the concept of SSD. This approach is designed to test necessary SSD conditions that must hold for SSD efficiency within traditional DEA models. As mentioned in the previous section, the SSD test for stock selection requires knowledge of the CDF of the stock returns and consists of pairwise comparisons of these CDFs. Obtaining these distributions is complicated and not feasible in many practical cases. For this reason a number of authors (Bawa et al., 1982; Post, 2003; Kuosmanen, 2004) have developed linear models that only test for the necessary conditions of SSD efficiency that help, at least, to remove those stocks that are not SSD efficient and are dominated by the other stocks. Lozano et al. used these linear models and adapted them as an inefficiency measure in their proposed DEA models. These DEA models are simpler than the SSD linear models mentioned above, both in terms of the number of constraints and the number of variables. Table 1 provides a summary of these DEA models. Four DEA models use risk measures as input and return measures as output while two models use return and safety measures as output with a constant input equal to unity. Each model should be solved for every stock to investigate its SSD relationship against other stocks and also against their combinations. If the value of the objective function of the model is greater than zero, then the assessed stock is not DEA and SSD efficient. However, a value of zero for the objective function implies that the assessed stock is DEA efficient and also a candidate for being SSD efficient. Therefore, these DEA models are applicable in cases where we want to remove the stocks that are not SSD efficient.

Table 1: Summary of DEA-SSD Models

DEA Model	Inputs	Outputs
MR-DASD	Downside absolute semi-deviation (DASD) which is the standard deviation of returns below the mean return value.	- Mean return (MR)
MR-MDUA	Constant equal to unity	- Mean return (MR) - Mean downside under-achievement (MDUA) which is the Difference between mean return (MR) and Downside absolute semi-deviation as safety measure (DASD)
MR-WADQ	Weighted absolute deviation from pth quantile (for any p from 0 to 1). In this model deviation above the pth quantile are given a weight of one while deviations below the quantile are weighted with (1-p)/p.	- Mean return (MR)
MR-TVaR	Constant equal to unity	- Mean return (MR) - Tail value at risk (TVAR)[1] which is the Difference between mean return (MR) and weighted absolute deviation from quantile (WADQ)
MR-BTSD	Below target semi-deviation (BTSD) which is the standard deviation of returns below the specified target value.	- Mean return (MR)
ATMR-BTSD	Below target semi-deviation (BTSD) which is the standard deviation of returns below the specified target value.	- Above target mean return (ATMR) which is the standard deviation of returns above the specified target value.

[1] Also known as conditional value-at-risk or worst conditional expectation

3.3 Proposed Approach

One of the shortcomings of the SSD statistical test is its inability to compare from a practical sense, the CDF of the return of a given stock to the CDFs of returns of all possible portfolio combinations of the other stocks. Therefore, the efficient set, i.e. the set of non-dominated stocks, may contain stocks with weak SSD efficiency. These weak SSD efficient stocks are those that although their CDFs are never dominated by others in individual comparisons can be dominated by a combination of other CDFs. Considering this shortcoming, the advantage of the aforementioned DEA model over the SSD statistical test is that not only can the former can compare each pair of stocks in the set, but it can also compare each stock with possible linear combinations of others. These comparisons are only based on necessary rules of SSD efficiency and thus the stocks selected by such DEA tests are not necessarily SSD efficient. Here we propose to integrate both approaches to generate an optimal stock selection method that overcomes the limitations of the SSD statistical test and the DEA approach when applied individually. In order to select the best stocks according to the SSD rule, first we test the hypothesis of the SSD against the alternative hypothesis of the non-SSD by statistical testing. Then we apply DEA tests on the new set of stocks that we generated by applying the SSD test (SSD-set). The necessary rules for SSD efficiency are incorporated as inputs and outputs in different DEA models. We have decided to use the most important conditions that are proposed by Lozano et al. in this study. Therefore, we have six DEA models that should be solved individually for our SSD-set (see Table1 for further details). This procedure helps us to remove stocks from the SSD-set that are dominated by the combination of other stocks in that set. We refer to stocks that we removed as stocks with weak SSD efficiency. By applying the proposed method, we narrowed down the SSD-set to strong SSD efficient stocks (SSD-DEA-set). Figures 1 to 3 summarize the SSD, DEA and the proposed approach along with their benefits and limitations. In the next section, we apply this approach to a set of financial stocks from an S&P index.

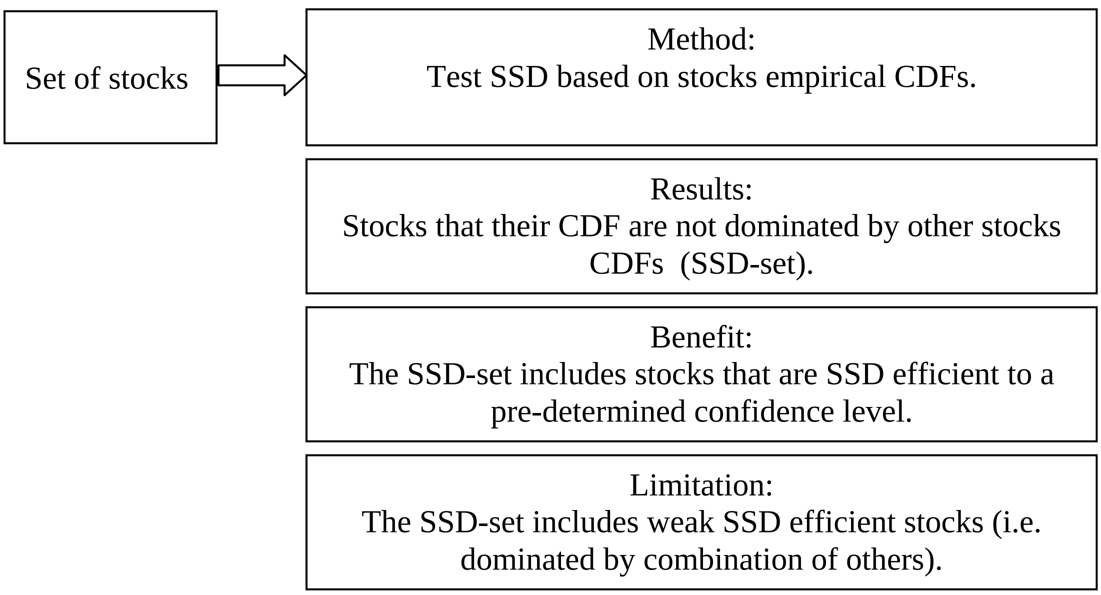

Figure 1: SSD statistical test

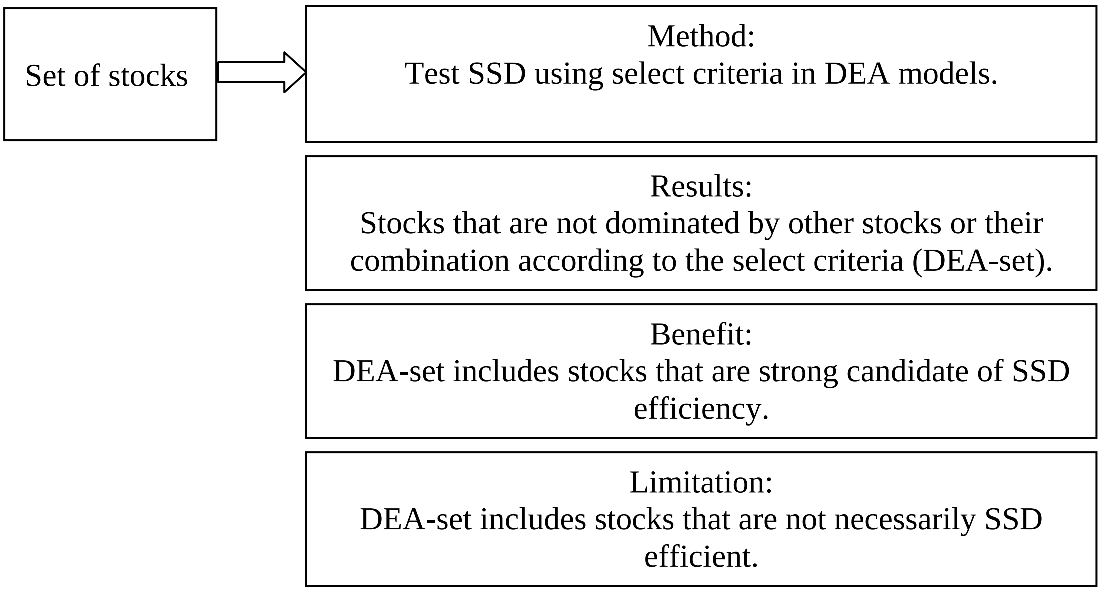

Figure 2: DEA-SSD test of Lozano et al.

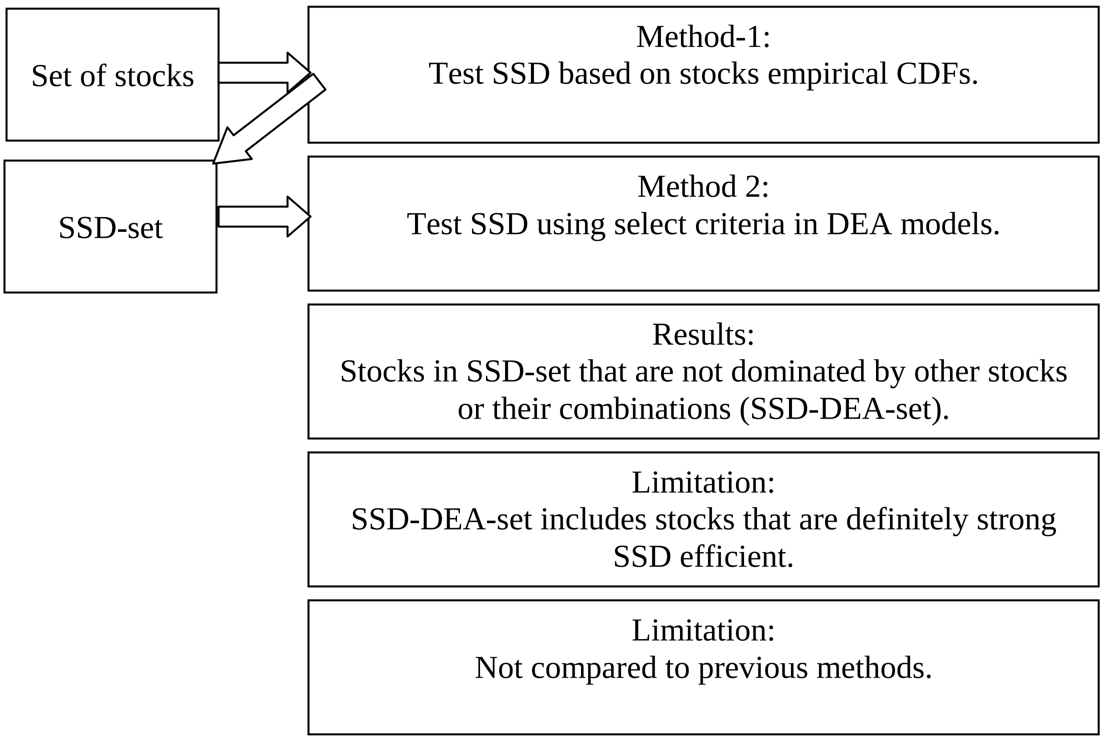

Figure 3: Proposed method.

4. Discussion of Findings

In this section we analyse the SSD relations among 68 financial stocks obtained from the S&P 500 Index. We have used data on the daily returns of these 68 stocks (labelled S1 to S68) between December 2000 and December 2009. Based on descriptive statistics of the stock returns, most of the standard deviations range from 30 to 81 percent. The majority (69%) of stocks have negatively skewed daily returns, i.e. the third moment of the stocks monthly returns distribution is negative. The kurtosis of the stock returns are significantly greater than the normal distribution, suggesting that all stock return distributions have fat tails, or, in other words, the probability of very large and of very small returns is higher than one would observe under a normal distribution.

In order to select the best stocks according to the SSD rule, first we test the hypothesis of the SSD against the alternative hypothesis of the non-SSD. Then we apply the necessary rules that must hold for SSD efficiency. These rules are incorporated in the DEA models. In this way, we remove the stocks that are weak SSD efficient, i.e. those stocks that are never dominated by others and reported as SSD efficient but are dominated by a combination of other stocks. The SSD test of Klaver (2006) is chosen to apply to the data. The results for the SSD test are reported in Table 2. The stocks are shown in rows as well as columns labelled from S1 to S68. Here the value of "1" indicates rejection of the dominance hypothesis when the stock in a row is compared to the stock of a given column and a value of "0" signifies no rejection. A low sum at the end of a row indicates that the particular stock in that row is not dominated by others very often. In the descriptive sense, SSD dominancy can be established in 1088 of 4556 comparisons between pairs of stocks. The efficient set is defined as the set of stocks which are not dominated by other stocks. In other words, the efficient set will consist of stocks with zero sum values in their corresponding rows. The analysis shows that six of the 68 stocks are SSD efficient: S19, S29, S30, S41, S44 and S46. The efficient set is still consists of stocks that weakly dominate others. The DEA models aim to compare each of the following, SSD efficient, stocks with the combination of the stocks in the set (according to the associated inputs and outputs, see Table 1 for details).

The weak SSD efficient stocks with higher inputs and lower outputs will be dominated by combinations of other stocks in the DEA models and will be removed from being strong SSD efficient. The remaining SSD efficient stocks are the ones that are both DEA and SSD efficient. In other words these stocks are never dominated by any individual stock or by a combination of stocks according to the necessary rules of dominancy. We will refer to this new set as stocks with strong SSD dominancy over others. Therefore, the efficient set is reduced from six stocks to only three stocks of S19, S41 and S44 that are reported as efficient by all six DEA models and the initial SSD test.

Table 2: SSD Test Result

Stocks	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10	S11	S12	S13	S14	S15	S16	S17	S18	S19	S20	S21	S22	S23	S24	S25	S26	S27	S28	S29	S30	S31	S32	S33	S34	S35	S36	S37	S38	S39	S40	S41	S42	S43	S44	S45	S46	S47	S48	S49	S50	S51	S52	S53	S54	S55	S56	S57	S58	S59	S60	S61	S62	S63	S64	S65	S66	S67	S68	Sum
S1	0	0	0	0	0	0	1	0	0	0	1	0	1	0	0	0	1	0	1	1	1	0	1	0	1	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1	1	1	1	0	0	1	0	0	0	1	1	0	0	1	0	0	0	0	0	0	0	0	0	1	20
S2	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	4
S3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3
S4	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	1	0	1	0	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	0	0	1	0	0	0	1	1	0	0	1	0	0	0	0	0	1	0	0	0	1	16
S5	0	0	0	0	0	0	1	0	0	0	1	0	0	0	0	0	1	0	1	0	1	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	0	0	1	0	0	0	1	1	0	0	1	0	0	0	0	0	0	0	0	0	1	16
S6	0	1	1	0	0	0	1	0	0	1	0	1	1	1	0	0	1	0	1	1	1	1	1	0	1	0	1	1	0	1	0	0	1	1	0	0	1	1	0	0	1	0	0	1	1	1	1	0	1	1	0	1	0	1	1	1	0	1	0	0	0	0	0	1	1	0	0	1	36
S7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0	4
S8	1	1	1	1	1	0	1	0	1	1	1	1	1	1	0	0	1	0	1	1	1	1	1	1	1	1	1	0	1	1	0	0	1	1	0	0	1	0	0	0	1	0	0	1	1	1	1	0	1	1	0	1	0	1	1	1	0	1	1	0	0	0	0	1	1	0	0	1	43
S9	0	0	0	0	0	0	1	0	0	0	1	0	0	0	0	0	1	0	1	1	1	1	1	1	1	0	0	0	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	1	1	1	1	0	0	1	0	0	0	1	1	0	0	1	0	0	0	0	0	1	0	0	0	1	22
S10	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	1	0	1	0	1	0	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	1	1	1	1	0	0	1	0	0	0	1	1	0	0	1	0	0	0	0	0	0	0	0	0	1	17
S11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	4
S12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	1	1	0	0	0	0	0	0	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0	8
S13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	4
S14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
S15	0	0	1	0	0	0	1	0	0	1	1	1	1	1	0	0	1	0	1	1	1	0	1	1	1	1	1	0	1	1	0	0	1	1	0	0	1	0	0	0	1	0	0	1	1	1	1	0	0	1	0	0	0	1	1	1	0	1	0	0	0	0	0	1	0	0	0	1	33
S16	1	1	1	1	1	0	1	0	1	1	1	1	1	1	0	0	1	0	1	1	1	1	1	1	1	1	1	1	1	1	0	0	1	1	0	0	1	1	0	1	1	0	0	1	1	1	1	0	1	1	0	1	0	1	1	1	0	1	1	0	0	0	0	1	1	0	0	1	46
S17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	4
S18	1	0	1	1	1	0	1	0	1	0	1	1	1	1	0	0	1	0	1	1	1	1	1	1	1	1	1	0	1	1	0	0	1	1	0	0	1	0	0	0	1	0	0	1	1	1	1	0	1	1	0	0	0	1	1	0	0	1	0	0	0	0	0	1	0	0	0	1	37
S19	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
S20	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	0	0	1	0	0	0	0	1	0	0	1	0	0	0	0	0	0	0	0	0	1	12
S21	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3
S22	0	0	0	0	0	0	1	0	0	0	1	0	0	0	0	0	1	0	1	1	1	0	1	1	1	1	0	0	1	1	0	0	0	0	0	0	1	0	0	0	1	0	0	1	1	1	1	0	0	1	0	0	0	1	1	0	0	1	0	0	0	0	0	1	0	0	0	1	24
S23	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
S24	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	1	1	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	0	0	1	0	0	0	0	1	0	0	1	0	0	0	0	0	0	0	0	0	1	14
S25	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
S26	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2
S27	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
S28	0	0	1	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	0	1	0	0	0	0	0	1	0	0	1	0	0	0	1	0	0	1	0	0	0	1	0	0	1	0	0	1	0	0	1	0	0	0	1	1	0	0	1	0	0	0	0	0	0	0	0	0	1	16
S29	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
S30	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
S31	1	0	1	1	1	0	1	0	1	1	1	1	1	1	0	0	1	0	1	1	1	1	1	1	1	1	1	1	1	1	0	0	1	1	0	0	1	0	0	1	1	0	0	1	1	1	1	0	1	1	0	1	0	1	1	1	0	1	0	0	0	0	0	1	1	0	0	1	43
S32	0	1	1	0	0	0	1	0	0	1	0	1	1	1	0	0	1	0	1	0	1	0	1	0	1	0	1	1	0	1	0	0	1	1	0	0	1	0	0	0	1	0	0	1	1	1	1	0	0	1	0	1	0	1	1	0	0	1	0	0	0	0	0	0	1	0	0	1	30
S33	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3
S34	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	6

Table 2: SSD Test Result (Continued)

Stocks	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10	S11	S12	S13	S14	S15	S16	S17	S18	S19	S20	S21	S22	S23	S24	S25	S26	S27	S28	S29	S30	S31	S32	S33	S34	S35	S36	S37	S38	S39	S40	S41	S42	S43	S44	S45	S46	S47	S48	S49	S50	S51	S52	S53	S54	S55	S56	S57	S58	S59	S60	S61	S62	S63	S64	S65	S66	S67	S68	Sum
S35	0	1	1	0	0	0	1	0	0	0	0	1	0	1	0	0	1	0	1	0	1	0	1	0	1	0	1	1	0	1	0	0	1	1	0	0	1	0	0	0	1	0	0	1	1	1	1	0	1	1	0	1	0	1	1	0	0	1	0	0	0	0	0	0	1	0	0	1	29
S36	0	1	1	0	0	0	1	0	0	1	0	1	1	1	0	0	1	0	1	0	1	0	1	0	1	0	1	0	0	1	0	0	1	1	0	0	1	0	0	0	1	0	0	1	1	1	1	0	0	1	0	1	0	1	1	0	0	1	0	0	0	0	0	0	0	0	0	1	28
S37	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	1	0	0	0	0	0	0	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0	7
S38	0	0	1	0	0	0	1	0	0	1	1	1	1	1	0	0	1	0	1	0	1	0	1	0	1	0	1	1	0	1	0	0	1	1	0	0	1	0	0	0	1	0	0	1	1	1	1	0	1	1	0	1	0	1	1	1	0	1	0	0	0	0	0	1	1	0	0	1	33
S39	0	0	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	0	0	1	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	8
S40	0	0	0	0	0	0	1	0	0	0	1	0	1	0	0	0	1	0	1	0	1	0	1	0	1	1	1	0	1	1	0	0	1	0	0	0	1	0	0	0	1	0	0	1	1	1	1	0	0	1	0	0	0	1	1	0	0	1	0	0	0	0	0	1	0	0	0	1	25
S41	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
S42	0	0	1	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	0	1	0	0	0	0	0	1	1	0	1	0	0	1	1	0	0	1	0	0	0	1	0	0	1	0	1	1	0	0	1	0	1	0	1	1	0	0	1	0	0	0	0	0	0	0	0	0	1	20
S43	0	1	1	0	0	0	1	0	0	1	0	1	1	1	0	0	1	0	1	1	1	0	1	0	1	0	1	1	0	1	0	0	1	1	0	0	1	0	0	0	1	0	0	1	1	1	1	0	1	1	0	1	0	1	1	1	0	1	0	0	0	0	0	0	1	0	0	1	33
S44	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
S45	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	3
S46	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
S47	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
S48	0	0	1	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	0	1	0	1	0	0	0	1	0	0	1	0	0	1	1	0	0	1	0	0	0	1	0	0	1	1	1	1	0	0	1	0	1	0	1	1	0	0	1	0	0	0	0	0	0	1	0	0	1	22
S49	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	1	0	0	1	0	0	0	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0	7
S50	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	4
S51	0	0	1	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	0	1	0	1	0	0	0	1	1	0	1	0	0	1	1	0	0	1	0	0	0	1	0	0	1	0	1	1	0	0	1	0	1	0	1	1	0	0	1	0	0	0	0	0	0	0	0	0	1	21
S52	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3
S53	0	0	1	0	1	0	1	0	0	1	1	1	1	1	0	0	1	0	1	1	1	0	1	1	1	1	1	0	1	1	0	0	1	1	0	0	1	0	0	0	1	0	0	1	1	1	1	0	1	1	0	1	0	1	1	1	0	1	0	0	0	0	0	1	1	0	0	1	37
S54	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	3
S55	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2
S56	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	0	1	0	1	0	1	0	1	0	0	1	0	0	1	1	0	0	1	0	0	0	1	0	0	1	1	1	1	0	0	1	0	0	0	1	1	0	0	1	0	0	0	0	0	0	0	0	0	1	20
S57	0	1	1	0	0	0	1	0	0	1	1	1	1	1	0	0	1	0	1	1	1	1	1	0	1	0	1	1	0	1	0	1	1	1	0	1	1	1	1	0	1	1	0	1	1	1	1	0	1	1	1	1	1	1	1	1	0	1	0	1	0	0	0	1	1	1	0	1	45
S58	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
S59	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	1	0	1	0	0	0	0	1	0	0	0	0	0	0	1	0	0	0	1	0	0	1	1	1	1	0	0	1	0	0	0	0	1	0	0	1	0	0	0	0	0	0	0	0	0	1	15
S60	0	0	1	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	1	1	0	0	1	0	0	0	1	0	0	1	0	1	1	0	0	0	0	1	0	1	1	0	0	1	0	0	0	0	0	0	0	0	0	1	17
S61	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	0	1	1	1	1	1	0	1	65
S62	0	1	1	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	0	1	0	1	0	0	0	1	1	0	1	0	0	1	1	0	0	1	0	0	0	1	0	0	1	1	1	1	0	0	1	0	1	0	1	1	0	0	1	0	0	0	0	0	0	0	0	0	1	23
S63	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	0	1	0	1	1	1	1	1	1	1	1	1	1	1	1	0	0	1	1	0	0	1	1	0	1	1	0	0	1	1	1	1	0	1	1	1	1	1	1	1	1	0	1	1	1	0	0	0	1	1	1	0	1	52
S64	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	1	0	0	0	0	0	0	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0	8
S65	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	1	1	0	0	1	0	0	0	1	0	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	10
S66	0	0	1	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	0	1	0	0	0	1	0	1	1	0	1	0	0	1	1	0	0	1	0	0	0	1	0	0	1	1	1	1	0	0	1	0	1	0	1	1	0	0	1	0	0	0	0	0	0	0	0	0	1	22
S67	0	1	1	0	0	0	1	0	0	1	1	1	1	1	0	0	1	0	1	1	1	0	1	0	1	0	1	1	0	1	0	0	1	1	1	0	1	1	1	0	1	1	0	1	1	1	1	1	1	1	1	1	0	1	1	1	0	1	1	1	0	1	0	1	1	1	0	1	45
S68	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	5

5. Conclusion

In this paper, we have investigated SSD relations among 68 financial stocks from the S&P 500 Index. The SSD rule is the desirable conditions that all risk averse, non-satiated investors seek. Due to the high frequency of false rejection of SSD by empirical distributions, several statistical tests have previously been developed for surveying the SSD relationship. However, in these tests, SSD is not vigorously confirmed and in most cases, weak dominance of some stocks over others cannot be rejected at high level of confidence (95% or higher). Thus, we have tested the necessary rules of SSD efficiency incorporated into DEA models to remove the stocks that are weak SSD efficient. Weak SSD efficiency refers to stocks that are never dominated by others and reported as SSD efficient, but they are dominated by a combination of other stocks given the necessary rules of SSD efficiency. Based on the empirical study of 68 financial stocks from the S&P 500 Index, there are six stocks which are SSD efficient by statistical testing and the efficient set is reduced to only three stocks by applying the DEA models to remove the weakly dominant stocks. These three stocks are desirable choices for investors who prefer having more wealth to less and are risk averse.

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