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Volatility spillover and time-varying conditional correlation between the European and US stock markets

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In light of globalization and trading technology innovations it seems that the financial market(s) exhibit an increasing co-movement trend. The knowledge of the interrelationship and volatility transmission between international assets would help financial practitioners make their investment decisions, and financial regulators control the financial contagion. This paper aims to examine volatility spillover effects and test time-varying correlations across four stock indices namely, CAC, DAX, FTSE100 and S&P500 spanning the period 5th January, 2004 to 1st October, 2009. Two multivariate generalized autoregressive conditional heteroskedasticity (MVGARCH) models, namely BEKK (Engle and Kroner, 1995) and DCC (Engle, 2002) have been adopted because of their abilities to capture the leptokurtic, autocorrelation features of financial time series. Moreover, they are known to proffer a time-varying variance-covariance matrix which reveals various pieces of information about volatility and correlation. Firstly, we find that volatility spillover effects widely exist between the European and US stock markets using the BEKK model. The UK stock market is the main volatility transmitter within the European stock market while the US one is the main exporter worldwide. Secondly, we test the presence of time-varying correlation among international equity market using the DCC model. The results indicate that correlations are not only conditional but also significantly time-varying. Furthermore the results also show that the time-varying conditional correlation follows a mean-reverting process in the DCC model; however, according to this analysis this is only valid for the European stock markets.

Field of research: Multivariate GARCH (BEKK, DCC), time-varying correlation, volatility spillover effects, mean-reversion

1. Introduction

The global extent of the recent crisis and the potentially damaging consequences of being affected by contagion continuously attract public attentions. In the last two decades~~,~~ financial disorder quickly spread from one economic unit to another generating the “contagion” phenomenon. Examples of this “contagion” include the Asian stock market crisis in 1997, the Russian default in 1998 and the subprime mortgage financial crisis of 2007–present. Each of these propagated and affected not only neighboring but also distant markets all over the world. A salient feature involved in this phenomenon is that markets tend to move more

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closely together especially when market become agitated. Moreover the analysis of financial market integration, co-movement and the degree of correlation between assets plays a vital role in many financial decisions for market participants such as international trading companies and financial institutions. For example, the increased correlation between international assets may diminish the diversity of an international investment portfolio. This might force market participants to seek relative independent assets to maintain the optimal portfolio selection. However the fact that the pronounced features of financial asset prices are well-recognized and documented by economists result in obstacles to researchers obtaining an accurate estimation of financial co-movement and correlations. These features include volatility clustering, leptokurtosis and time-varying characteristics. Therefore there is a need for us to investigate the volatility spillovers and dynamic conditional correlations by using advanced econometric approaches.

There is an abundance of literature focusing on correlation and co-movements in the international financial markets using different advanced econometric approaches, see Embrechts, McNeil and Straumann (2001); Skintzi and Refenes (2006); Aslanidis, Osborn and Sensier (2008); Evans and McMillan (2009); Sun, Rachev, Fobozzi and Kalev (2009) *etc*. This paper explores the dynamic linkages between the European and US stock markets to answer and test the following research questions and hypothesis: 1) whether volatility spillover effects exist between the European market and US markets? 2) Do we have a symmetric or asymmetric volatility mechanism among the world stock markets? 3) Who is (are) the main volatility transmitter(s) during the past five years? 4) How the time-varying conditional correlations differ from unconditional correlation in terms of magnitude and direction? 5) Are the time varying conditional correlation between stock index return series mean-reverting? The pilot study has been carried out and seen in the authors forthcoming publication (Xiao and Dhesi, (2010)).

This paper is structured as follows: Section 2 is a critical literature review addressing existing research methodologies and previous empirical research results. Section 3 presents the description of main methodologies adopted in this paper namely, BEKK and DCC multivariate GARCH models. In section 4 data description and analysis are discussed. Empirical studies results and discussion are shown in section 5. Section 6 provides conclusions and answers to our research questions. Future work and some remarks relating to this study also are given in section 6.

2. Literature Review

The liberalization of capital flows promoted by trading technologies innovation and transparency of news has resulted in the increased return and volatility transmission between international financial markets. Transmission mechanisms presenting between returns and volatilities play a critical role in examining the distribution and interdependence across international financial markets. This is due to the following explicit and implicit reasons summarized by Harris and Pisedtasalasai (2005): firstly transmission mechanism is an indicator of market efficiency. If spillovers are found in return series then it is possible to exploit strategy profits which are against the market efficiency criteria. Secondly, it is acknowledged that information of return spillover effects is helpful to allocate assets and to construct portfolios. Thirdly understanding of volatility spillovers is crucial when dealing with those financial applications requiring for estimation of conditional volatility such as derivations pricing and value at risk (VaR) estimation. Interesting empirical studies contributions on examining volatility spillovers effects could be reviewed in Kanas (1998, 2002), Billio and Pelizzon(2003), Christiansen (2003.2004), Worthington and Higgsy (2004), Yang and Doong (2004), Lee (2006), Ozun (2007), Giannellis, Papadopoulos and Kanas(2008), Koulakiotis, Dasilas and Papasyriopoulos (2009), Tanizaki and Hamori (2009), Sun and Zhang(2009) etc. Koutmos and Booth (1995) examine the spillover effects among the New York, Tokyo and London stock markets and shows the transmission of volatility is asymmetric and is more pronounced when the news is bad and coming from either US or UK market. Kanas (1998) studies on the transmission effects among the London, Paris and Frankfurt stock markets and concludes that returns and innovation~~s~~ spillovers are higher during the post-crash time. Billio and Pelizzon (2003) obtain evidence showing that volatility spillover from the world index return series have increased after the introduction of the EMU (European Monetary Union) for most European stock market. Christiansen (2007) investigates volatility spillover from the US and aggregate European asset markets into European national asset markets with the innovation of incorporating the bond markets into analysis. This study reports that significant volatility spillover effects widely exist and that the national bond and stock markets reciprocally influence each other. Koulakiotis, Dasilas and Papasyriopoulos (2009) find evidence of volatility and error transmission spillover effects from three European financial regions and claim that each region has its own main exporter. Among the numerous literatures, two types of multivariate GARCH model are extremely popular. Some researchers are using a multivariate extension of Nelson (1991) univariate exponential GARCH (EGARCH) which allows for time-varying correlation. Whereas, some academicians prefer the BEKK (Engle and Kroner, 1995) model which is famous for the superiority and flexibility of modeling spillover effects for low dimensions (Sheppard (2003), Alexander (2008)).

In addition to volatility spillover effects we look into conditional correlation as the second implement to study the dynamic linkages between the European and US markets. The simple and traditional method of measuring dependence is linear correlation (namely Pearson’s correlation coefficient) because of its metric property of measuring dependence in multivariate normal distributions and, more generally, in multivariate spherical and elliptical distributions. One more explicit reason is because that this linear correlation measure is straightforward to calculate (Embrechts, McNeil and Straumann (1999)). However, the use of linear correlation is not unconditional; it is only a symmetric, linear dependence metric.

Furthermore, it is now well acknowledged that correlations of returns across international stock markets are not only strong, but also time-varying (Aslanidis, Osborn and Sensier, 2008). Therefore a better understanding is provided by varieties of multivariate generalized autoregressive conditional heteroskedasticity (GARCH) or stochastic volatility to measure the interdependence structure across financial markets. These types of more advanced method have been extensively investigated in the financial econometric literature and are used by a few sophisticated practitioners. A large number of financial tasks such as risk management, derivative pricing, hedging ratio estimation, value at risk (VaR) market making and portfolio selection could be accomplished by MVGARCH which have been evolved from the standard univariate GARCH model.

Bollerslev *et al* (1988) originally proposed the basic framework of MVGARCH which extends the univariate GARCH into the vectorized conditional-variance matrix. This VECH model involves a large number of parameters estimation（No. of parameters: $P = k^4/2 + k^3 + k^2 + k/2 = O(k^4)$, k is the number of dimensions）. In order to make estimation more tractable, Bollerslev *et al* (1988) proposed the diagonal VECH model. However, this type of MVGARCH model could not be used to examine spillover effects since it simplified the correlation between parameters. The factor GARCH (Engle *et al* 1990) reduces the number of parameters to $O(k^2)$, but empirical studies reveal its poor performance on low and negative correlations. Bollerslev (1990) proposed the constant correlation (CC) model; although it still allows volatility time-varying the conditional correlations are restricted to be time-invariant. Tsui and Yu (1999) have found out that the constant correlation assumption can be rejected for certain assets which indicate that the CC model may not be generous enough. Engle and Kroner (1995) made improvements based on the work of Baba, Engle, Kraft and Kroner and created a general quadratic form for the covariance equation which successfully eliminated the positive definiteness problem of the original VECH model. In the fully general BEKK model, the number of parameters needing to be estimated is $O(k^4)$, the standard BEKK estimation will involve $O(k^2)$ parameters. Other more plausible formulations of BEKK model include diagonal and scalar BEKK where the parameters are restricted to be either diagonal matrices or to be scalars. The most obvious shortcoming of those simplified BEKK models is that some information such as volatility spillover effects are missing in the variance-covariance matrix since the parameters have been reduced. Alexander (2000) demonstrates how to apply factor (or Orthogonal) GARCH models which limit the factors accounting for the amount of volatility. The most attractive feature of this kind of MVGARCH model is generous enough to provide a method for estimating any variance-covariance matrix using univariate GARCH models. However, Sheppard (2003) criticizes this approach in that it is hard to interpret the coefficients on the univariate GARCH model and that it performs poorly for less correlated systems such as individual equities since it reduce the number of parameter to O(k). In light of the pre-mentioned limitations of various MVGARCH models, Engle (2002) advocate a new class of MVARCH model which is named as dynamic conditional correlation (DCC). Intuitively the DCC model maintains the plausibility of the CC model whilst still allowing for time-varying conditional correlation. Sheppard (2003) makes a great contribution to the DCC model estimation by reducing the estimation of MVGARCH to a series of univariate GARCH process plus an additional correlation estimator. The specification of the univariate GARCH is generous to any GARCH process with normally distributed that satisfies the non-negative constraints and the stationary condition. This recent development is motivated by the usual phenomenon in multivariate modelling of the unequal or mismatching durations of different datasets. Patton (2006) proposed two maximum likelihood estimators (MLEs) of parameters of a multivariate model for time series with histories of different lengths.

Some interesting applications of multivariate GARCH model can be seen in work of Engle (2002), Sheppard (2003), Cappiello Engle and Sheppard (2006) and Tse and Tsui (2009). Some researchers choose BEKK such as Tastan (2006) to model time-varying correlation. However, the large number of parameters needing to be estimated for the high dimension general form has been emphasized and the exact interpretation and impact of the individual coefficients is difficult to discern and interpret (Sheppard, 2003). There is another strand of interesting study and correspondingly empirical studies using Enlge (2002) dynamic conditional correlation model (DCC) which is a generous model allowing for time-varying correlation as well as for the plausibility of estimation. Lee (2006) employs the DCC model to document the fact that the overall price level tended to move in the same direction as output in the periods before the World War II but in the opposite direction after the war. Aslanidis, Osborn and Sensier (2008) provide evidence on the source of co-movement in monthly US and UK stock returns by investigating the role of macroeconomic and financial variables with the DCC and related modified DCC models. Compared with the BEKK model, the prominent strength of the DCC model is that it does not suffer dimension hindrance and could be applied to any dimension. This is because the estimation can be decomposed into two steps: firstly estimating the univariate GARCH and subsequently constructing a maximum likelihood function which has only two parameters (details seen in section 3 Methodologies). However the DCC model imposes more restrictions on the type of dynamic effects than the BEKK model. In particular the conditional variance of returns only depends on the past squared returns, some of which can cause the ‘volatility spillovers’ to be excluded. Similarly, feedback from past volatilities or squared returns on correlations is severely limited in the DCC model. As a result, we choose the BEKK model to capture the volatility spillover effects and the DCC model to measure the dynamic conditional correlations.

3. Methodologies

3.1BEKK

In our study we use autoregressive moving average model ARMA (1, 1) to define the conditional mean of returns. Thus according to Baba, Engle, Kraft and Kroner (1995), the ARMA (1, 1)-BEKK (1, 1) model takes the following form:

$$r_{it} = \alpha + \varphi_{i,t-1} r_{i,t-1} + \varepsilon_{it} + \theta \varepsilon_{i,t-1} \quad i = 1,2,3,4 \tag{1}$$

$$\varepsilon_t \left| \Omega_{t-1} \sim N(0, H_t), \right.$$

$$H_t = C_0^{'} \; C_0 + A_{11}^{'} \varepsilon_{t-1} \varepsilon_{t-1}^{'} A_{11} + G_{11}^{'} H_{t-1} G_{11}, \tag{2}$$

Where r_t is an T by 1 vector of asset returns, εt is the innovation term in the return equation, Ω_{t-1} is the matrix of conditional previous information set and H_t is the variance-covariance matrix of the residuals term from Eq. (1) and it's guaranteed to be positive because the BEKK model uses a quadratic form for the parameter matrices to ensure a positive definite variance- covariance matrix. The parameter vector consists of elements of C which is a lower triangular matrix; $n \times n$ matrix A_{11} is showing ARCH effects and $n \times n$ matrix G_{11} which reveal the GARCH effects. The diagonal elements in the parameter matrix G measures the effect of lagged volatility; the off-diagonal elements capture the cross market effects (Zhang, 2009). Therefore, the BEKK model is very desirable to examine volatility spillover effects.

The log-likelihood function of the BEKK model is given by

$$L(\Theta) = \frac{-Tn}{2} + \ln(2\pi) - \frac{1}{2} \sum_{t=1}^{T} (\ln |H_t|) + \varepsilon_t^{'} \left| H_t^{-1} \right| \varepsilon_t) \tag{3}$$

Where n is the number of variables in the model, T is total number of the observation and θ is the vectors of unknown parameters need to be estimated.

3.2 DCC (1, 1)-GARCH (1, 1)

The DCC model assumes that return from k assets is conditionally multivariate normal with zero mean and covariance matrix H_t. The returns can be either zero mean or the residuals from a filtered time series.

$$r_t \left| F_{t-1} \sim N(0, Ht) \right. \tag{4}$$

and $Ht = D_t * R_t * D_t$

(5)

where D_t is the $k \times k$diagonal matrix of time varying standard deviations from univariate GARCH models with $\sqrt{hit}$ on the i^{th} diagonal. R_t is the time varying correlation matrix.

The log-likelihood of estimator is written as,

$$L = -\frac{1}{2}\sum_{t=1}^{T}(k\log(2\pi) + 2\log(|D_t|) + \log(|R_t| + \varepsilon_t^{'} R_t^{-1}\varepsilon_t) \quad (6)$$

where $\varepsilon_t \sim N(0,R_t)$ are the residuals standardized by their conditional standard deviations.

Engle and Sheppard (2001) propose to rewrite the elements D_t as multivariate GARCH in the following manner:

$$h_{it} = \omega_i + \sum_{p=1}^{p_i}\alpha_{ip}\varepsilon_{it-p}^2 + \sum_{q=1}^{Q_I}\beta_{iq}h_{it-q} \quad (7)$$

for $i=1,1,\ldots k$ with usual GARCH restrictions for non-negativity and stationary being imposed: $\sum_{p=1}^{p_i}\alpha_{ip} + \sum_{q=1}^{Q_I}\beta_{iq} < 1$

The second component of the framework consists of a specific DCC (M, N) structure, which can be expressed as:

$$R_t = Q_t^{*-1}Q_tQ_t^{*-1} \quad (8)$$

where the proposed dynamic correlation:

$$Q_t = (1 - \sum_{M=1}^{M}a_m - \sum_{n-1}^{N}b_n)\overline{Q} + \sum_{m=1}^{M}a_m(\varepsilon_{t-m}\varepsilon'_{t-m}) + \sum_{n=1}^{N}b_nQ_{t-n} \quad (9)$$

Where Q_t is the conditional variance-covariance matrix of residuals with its unconditional (time-invariant) variance-covariance matrix $\overline{Q}$ resulting from Eq. (6). Q^*_t =diag {$\sqrt{qii}$}

Especially, Engle (2002) specifies the DCC model through the GARCH (1, 1) - type process

$$q_{ij,t} = \overline{\rho_{ij}}(1-\alpha-\beta) + \alpha\varepsilon_{i,t-1}\varepsilon_{j,t-1} + \beta q_{ij,t-1} \quad i, j=1,2$$

(10)

Where $\overline{\rho_{12}}$ is the assumed constant correlation between $\varepsilon_{1,t}$ and $\varepsilon_{2,t}$, α is the new coefficient and β is the decay coefficient. The model will be mean-reverting if $\alpha+\beta<1$.

The quantity $q_{12,t}$ from the above equation is normalized using

$$\rho_t = \frac{q_{12,t}}{\sqrt{(q_{11,t}, q_{22,t})}} \quad (11)$$

This value including the sign is our main interesting results which represent the conditional correlation between different equity markets.

4. Data analysis

The daily closed prices of FTSE100, CAC, DAX and S&P500[i] stock indices were obtained from Thomason DataStream over the period 5th Jan, 2004 to 1st Oct, 2009. There are slight difference between different stock markets trading days of each year, especially the end of each month and the public holidays. Therefore, we select data from common trading days of the four indices and delete the date when at least in one country is on holiday and accordingly the final samples consist of 1340*4 data items. Daily return of stock market (as a percentage) is defined as:

$$\gamma = 100(\ln P_t - \ln P_{t-1}) \tag{12}$$

where P_t is the daily closed price at time period t.

Table 1 shows the basic descriptive statistics. FTSE100, CAC, DAX index return series display mild positive skewness while the S&P500 is negatively skewed. All the return series display significant leptokurtic behaviour as evidenced by large kurtosis with respect to the Gaussian distribution.

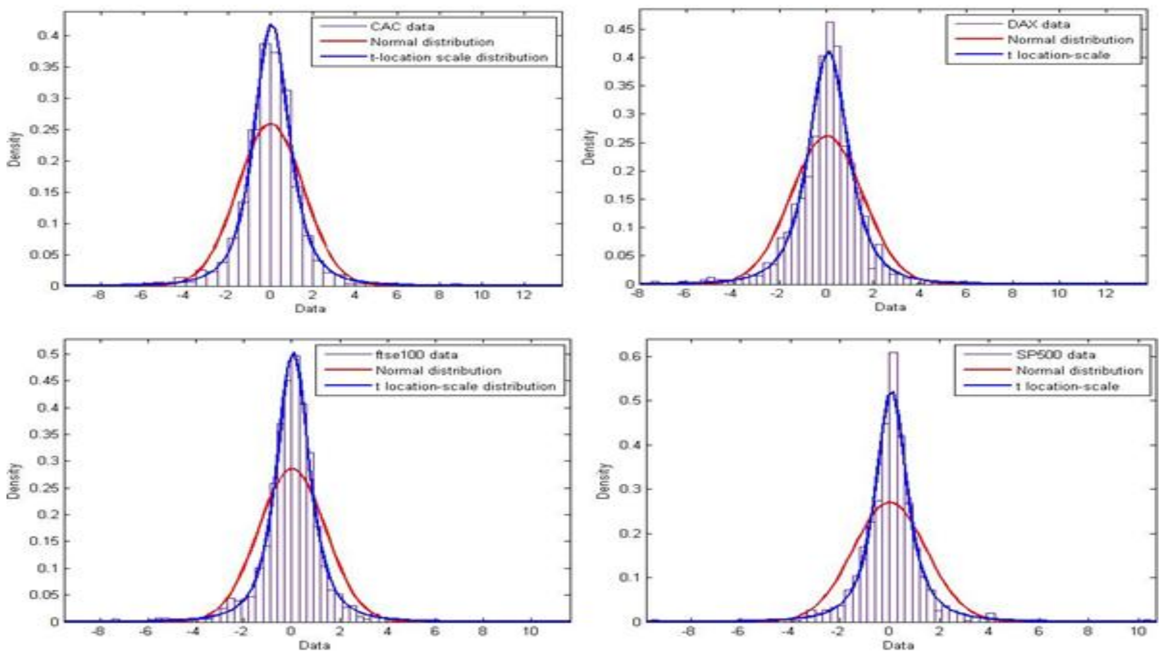
Table 1:Summary Statistics

	CAC	DAX	FTSE100	S&P500
Mean	0.0023	0.0238	0.0084	-0.0064
Median	0.0377	0.1079	0.0541	0.0788
Maximum	13.3048	13.4627	11.1112	10.4236
Minimum	-9.4715	-7.7391	-9.2646	-9.4695
Std. Dev.	1.5433	1.5264	1.3960	1.4766
Skewness	0.3776	0.3709	0.1230	-0.2915
Kurtosis	13.9873	13.5211	14.4281	13.2303
Jarque-Bera	6772.072[0.0000]	6211.102[0.0000]	7295.278[0.0000]	5862.42[0.0000]
LB-Q(4)	31.87 [0.20e-5]	16.18 [0.0028]	35.37[0.39e-6]	34.08[0.72e-6]
LB-Q(12)	76.77 [0.00e-5]	29.00[0.0039]	89.56[0.00e-6]	54.34[0.24e-6]
LB-Q² (4)	360.33 [0.00]	260.96 [0.00]	547 [0.00]	547.6 [0.00]
LB-Q²(12)	935.33 [0.00]	669.65 [0.00]	1176.3 [0.00]	1920.7 [0.00]
Unconditional correlations	ρ (CAC/DAX) 0.921936	ρ(CAC/FTSE100) 0.934918	ρ (CAC/S&P500) 0.5865	
	ρ(/DAX/FTSE100) 0.8665	ρ (DAX/S&P500) 0.6179	ρ(FTSE100/S&P500) 0.5756	

Notes: p values are in square brackets. Jarque-Bera tests normality distribution of return series and defined as T = [(s²)/6] + (k-3)²/24)], where *T, S* and *K* denotes sample size, skweness and kurtosis. *LB-Q(p)* is the statistics of Ljung-Box Q test which test for the null hypothesis that there is no autocorrelation in return series at lag *p*. *LB-Q² (p)* is the same test to detect any departure from the white noise behavior for squared return series.

The Jarque-Bera test statistics show that all the return series depart from the null hypothesis of normality distribution. Visualized evidence could be seen in Figure1 that none of these stock indexes are normally distributed. The Ljung-Box tests for lag 4 and lag12, calculated for both return and squared return series reveal that returns are strongly autocorrelated for all the retained indices.

Figure1 Probability Density Functions (*PFD*) fitting



Note: Apparently all the return series travel away from the normal distribution.

5. Empirical studies and discussion

5.1 Volatility spillover effects analysis

Table 2 summarizes the estimation results of bivariate N-BEKK for the European indices in a pairwise manner. The matrices *A* and *G* are useful in examining the relationship in terms of volatility in Eq. (2). The diagonal elements in matrix *A* report an ARCH effect, while the diagonal elements in matrix *G* capture a GARCH effect (also known as volatility persistence). As shown in Table 2, all the a_{11}, a_{22}, g_{11} and g_{22} are statistically significantly at 5% significance level which implies a strong ARCH and GARCH (1, 1) processes determining conditional variances of each stock index series. In other words, the conditional variance of CAC, DAX and FTSE100 index return series are strongly affected by their own past shocks and volatility effects. Additionally all the stock exchange markets present high volatility persistence. For example 0.939 for CAC index in CAC/DAX means that nearly 94% of volatility of the previous day persists the next day.

Table 2: Bivariate BEKK estimation results within the European stock markets

	CAC/DAX		CAC/FTSE100		DAX/FTSE100	
Parameter	Estimate	Standard error	Estimate	Standard error	Estimate	Standard error
c_{11}	0.1571	0.0013	0.1960	0.0008	0.1905	0.0021
c_{21}	0.1832	0.0059	0.1190	0.0030	0.1143	0.0008
c_{22}	0.0788	0.0044	-0.0176	0.0018	0.0495	0.0003
a_{11}	0.3053	0.0328	0.1944	0.0576	0.3140	0.0169
a_{12}	0.0004	0.0384	0.0177	0.0364	0.0735	0.0032
a_{21}	-0.0028	0.0145	0.0317	0.0780	0.0395	0.0226
a_{22}	0.3165	0.0249	0.2639	0.5041	0.0169	0.0061
g_{11}	0.9390	0.0249	0.8780	0.0259	0.9467	0.0030
g_{12}	-0.0019	0.0249	-0.0824	0.0148	-0.0153	0.0010
g_{21}	0.0087	0.0194	0.1031	0.0365	-0.0208	0.0046
g_{22}	0.9406	0.0147	1.0381	0.0221	0.9520	0.0017

Note: The c_{ij} a_{ij} and g_{ij} are the estimates of the elements of C_0, A_{11} and G_{11} matrices in Eq. (2), respectively.

The off-diagonal elements of matrices *A* and *G* measure the cross-market effects such as shock and volatility spillover effects among the three stock markets. First of all, according to the reported statistics we claim that bilateral shock and volatility spillover effects exist in CAC/FSTE100 and DAX/FTSE100 pairwise but not in pairwise CAC/DAX which implies that volatilities on both the French and Germany stock markets are influenced by UK market volatility transmission. Secondly, it is consistent with previous literature in that the volatility spillover effects are not symmetric and evidence demonstrates that the UK market is the main transmitter within the European markets. In particular, g_{12} (-0.0824) and g_{21} (0.1031) in CAC/FTSE100 pairwise, indicates the level of the volatility transmission. Volatility transmission from UK to France is 10.31% which implies that a 1% increase in returns of the FTSE100 index transmits 10.31% volatility to CAC index. Regarding volatility spillover effects from CAC to UK, it is only 8.24%. Volatility transmission between the German and UK market is comparative lower with absolute value of 1.53% from the German to UK and 2.08% on the other way round which reveals that the German and the UK stock markets are more independent. Last but not least, in Table 3, there is a very similar shock transmission pattern with volatility transmission within the European stock markets.

Table 3 documents the spillover effects between the three listed European stock markets and US stock market. All the estimates, with the exception of C_{22}, are statistically significant at 5% significant. The insignificance of C_{22} does not matter since it is just a constant in Eq. (2). Again, significant ARCH effects and GARCH persistence are found. Interestingly but perhaps not surprisingly, the major transmitter in the worldwide s the S&P500. This is due to the fact that volatility spillover effects exported from US to European stock markets are double or nearly triple that in the opposite direction. For example, the S&P500 index transmits 29.39% volatility spillover to the DAX index while only about half that magnitude 14.08% the way around. Volatility spillover transmit from the S&P500 index to the CAC index is 21.43% and the FTSE100 index imported about 9.74% volatility spillover effects from US and exports only 3.42%.

Table 3: Bivariate BEKK estimation results between the European and US stock markets

	CAC/S&P500		DAX/S&P500		S&P500/FTSE100	
	Estimates	Standard error	Estimate	Standard error	Estimate	Standard error
c_{11}	0.3287	0.0031	0.4655	0.0047	0.0653	0.0017
c_{21}	0.0534	0.0013	0.0863	0.0016	-0.1724	0.0011
c_{22}	-3.57E-05	0.0003	-0.00001	0.00002	0.0005	0.0019
a_{11}	0.4331	0.0032	0.3911	0.0046	-0.0382	0.0054
a_{12}	0.2038	0.0018	0.0802	0.0019	-0.4373	0.0036
a_{21}	-0.6007	0.0076	-0.5422	0.0061	0.2407	0.0026
a_{22}	-0.0594	0.0040	0.0764	0.0026	0.3308	0.0033
g_{11}	0.7304	0.0056	0.6621	0.0032	0.9923	0.0005
g_{12}	-0.1317	0.0028	-0.1408	0.0018	0.0974	0.0006
g_{21}	0.2143	0.0039	0.2939	0.0021	-0.0342	0.0019
g_{22}	1.0575	0.0009	1.0691	0.0006	0.8534	0.0012

Note: The c_{ij} a_{ij} and g_{ij} are the estimates of the elements of C_0, A_{11} and G_{11} matrices in Eq. (2), respectively. The italic values following the estimated parameters are the corresponding standard errors.

5.2 Time-varying conditional correlation analysis

DCC (1, 1)-GARCH (1, 1) is adopted to examine the time varying conditional correlation between European stock market and US in this section. Table 4 displays estimation results for the DCC (1, 1) model for the three European stock indices pairwise. At first sight, the sums of α_{ii} and β_{ii} are fairly close to 1, indicating rather high persistence in conditional variances. Moreover, the mean values of conditional correlation coefficients $\overline{\rho_{12}}$ which reflect unconditional correlation are very high (0.9934, 0.8824 and 0.8473 for the three European pairwise, respectively). However, the statistically significant a_1, b_1 at 5% significance level offer evidence against constant correlation which implies that the DCC model is favourable compared with the CC model. Furthermore, in the bivariate DCC (1, 1) estimation, the sums of a_1 and b_1 are equals to 0.75, 0.9783 and 0.9118 respectively. Therefore, $a_1+b_1<1$ proves the process described by the model is said to be mean reverting. The implication behind this is that after a shock occurs, the correlations will in time return to the long-run unconditional level.

Table 4: Bivariate DCC(1,1) estimation results within the European stock markets

	CAC/DAX		CAC/FTSE100		DAX/FTSE100	
	Estimate	Standard Error	Estimate	Standard Error	Estimate	Standard Error
ω_1	0.0252	0.0001	0.0252	0.0001	0.0301	0.0001
ω_2	0.0996	0.0004	0.0996	0.0004	0.1015	0.0001
α_{11}	0.8870	0.0004	0.8870	0.0004	0.8845	0.0004
α_{22}	0.0301	0.0001	0.0125	0.00003	0.0125	0.00003
β_{11}	0.1015	0.0005	0.1055	0.0004	0.1055	0.00003
β_{22}	0.8845	0.0004	0.8882	0.0004	0.8882	0.0004
a_1	0.0949	0.0008	0.0698	0.0002	0.0714	0.0005
b_1	0.6451	0.0063	0.9085	0.0004	0.8404	0.0005
$\bar{\rho}_{12}$	0.9334		0.8824		0.8473	
Log-likelihood	-2849.2090		-2871.2642		-3157.5742	
AIC	-5682.4179		-5726.5285		-6299.1485	
BIC	-5604.1336		-5684.9311		-6257.5510	

Note: ω, α, β are the estimates in Eq. (7) and log-likelihoods are calculated from Eq. (6). ρ_{12} is calculated from Eq. (11). AIC= (2*log-likelihood) + (2k) and BIC= (2*log-likelihood) + (k*log (T)) where k is the number of parameters in the model and T is the number of observations. In this study, T equals to 1339 for each series and k is 11, 12 for bivairate N-BEKK and T-BEKK, respectively. The model yields the lowest ACI (BIC) value (including negative sign) is considered to generate data best.

Table 5 reports DCC (1, 1) estimation results for CAC/S&P500, DAX/S&P500 and FTSE100/S&P500 pairwise to examine the conditional correlation. Again, the sum of α_{ii} and β_{ii} for each univariate GARCH estimation is fairly close to 1 which shows the high persistence of conditional volatility. The unconditional correlations (0.5563, 0.5657, and 0.5222, respectively) are not as high as those presented within the pairwise European market. Compared with the European pairwise sums, the sum of a_1 and b_1 in Table 5 is fairly close to 1 for all pairwise estimation which reminds us that changes in conditional correlation can last for long time. We claim that within the European stock markets, the conditional correlations are following a mean-reverting process while shocks from the US market are not transitory; they can persist for a long term period.

Table 5: Bivariate DCC(1,1) estimation results between the European and US stock markets

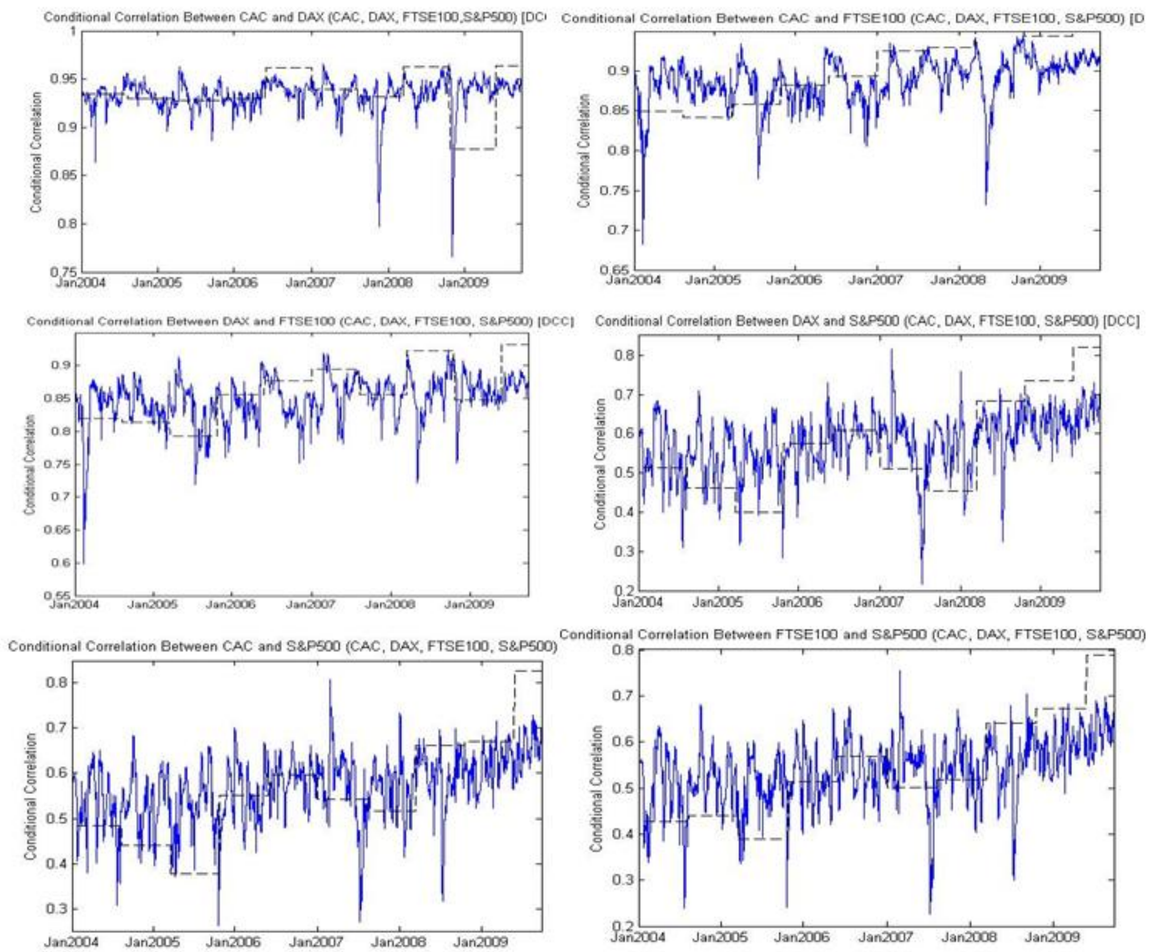
	CAC/S&P500		DAX/S&P500		FTSE100/S&P500	
	Estimate	Standard Error	Estimate	Standard Error	Estimate	Standard Error
ω_1	0.0252	0.0001	0.0301	0.0001	0.0125	0.00003
ω_2	0.0996	0.0004	0.1015	0.0005	0.1055	0.00044
α_{11}	0.8870	0.0004	0.8845	0.0004	0.8882	0.00044
α_{22}	0.0145	0.0001	0.0145	0.0001	0.0145	0.00005
β_{11}	0.0786	1.77E-04	0.07863	0.00018	0.0786	0.00018
β_{22}	0.9101	0.0002	0.91014	0.00018	0.9101	0.00022
a_1	0.0160	0.00003	0.0134	0.00003	0.0125	0.00002
b_1	0.9820	0.00003	0.9835	0.00003	0.9865	0.00002
$\overline{\rho_{12}}$	0.5563		0.5657		0.5222	
Log-likelihood	-3749.3503		-3790.8953		-3565.0764	
AIC	-7482.7006		-7565.7906		-7114.1527	
BIC	-7441.1032		-7524.1932		-7072.5553	

Note: ω, α, β are the estimates in Eq. (7) and log-likelihoods are calculated from Eq. (6). ρ_{12} is calculated from Eq. (11). AIC= (2*log-likelihood) + (2k) and BIC= (2*log-likelihood) + (k*log (T)) where k is the number of parameters in the model and T is the number of observations. In this study, T equals to 1339 for each series and k is 11, 12 for bivairate N-BEKK and T-BEKK, respectively. The model yields the lowest ACI (BIC) value (including negative sign) is considered to generate data best.

5.3 Graphic representation of DCC (1, 1)

The dynamic conditional correlations implied in the DCC model are plotted in Figure 2 as seen in blue curves. Without exception, the figures are representing time-varying patterns in correlation dynamic path. Accordingly, our strategy of using the DCC model is vindicated. In general the French, Germany and UK stock indices exhibit relative high conditional correlation which is consistent with unconditional correlation. The CAC and DAX indices have the highest correlation for returns between Jan, 2004 and Oct-2009. These figures also offer visualized evidence showing the conditional correlation obtained from the DCC(1,) model versus actual semi-annual sample correlation (in dash lines). By visually evaluating these figures, it is apparent that forecasting from the DCC (1, 1) is too noisy to represent the true correlation. In particular, the fluctuations often exceed long run changes in sample correlations. Last but not least, from the implied time-varying conditional correlation in DCC (1, 1), it is easy to distinguish dynamic conditional correlation and unconditional correlations between the European and US stock markets showing in Table1.

Figure2 Time-varying conditional correlation implied in DCC (1, 1)



6. Conclusion

This paper explores the dynamic linkages between the European and US stock markets from two aspects: volatility spillover effects and time-varying correlation. The basic statistic descriptive and preliminary tests guide us to employ ARCH/GARCH type models. In particular, we chose the BEKK (1,1) and DCC (1,1)-GARCH (1,1) to accomplish our empirical study based on the critical review of different type of multivariate GARCH models. Findings and discussion are as follows: there are significant and asymmetric volatility spillover effects exhibited in the international equity market. The UK market is the main transmitter within the European market while the S&P500 dominates the volatility transmission between the European and US stock markets from Jan, 2004 to Oct. 2009. Evidence is found to prove that there is mean-reverting process in the time-varying conditional correlation among the European stock markets. In contrast, the shock in correlation from the US stock market tend to persist in European markets for a long period. Though both unconditional and conditional correlation reveals that European stock markets are more dependent on each other. In light of the influential volatility spillover from US to European markets and the strong persistence of changes in the correlation between the US and European markets we may infer that there is missing information about interdependency between the European and US markets. It might be non-linear, tail dependence which could be captured by using copula functions[ii]. This limitation will be addressed by investigating the dynamic linkages between the European and US stock markets using Copula functions in our future works.

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[i] Financial times stock index (FTSE100), CAC for French Cotation Automatique continue index, Deutsche Aktien Index (DAX) and Standard and Poor500 (SP500).
[ii] Copulas are multivariate dimension functions to joint univariate marginal distribution together. Please see Nelson (1999) *Introduction to Copulas* and Cherubini Luciano and.Vecchiato (2004) *Copula Methods in Finance*

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