Global Economy and Finance Journal
Vol. 5. No. 2. September 2012. Pp. 42 – 57

A Structural Model of Corporate Debt with Stochastic Volatility

Xiaofei Li*

This paper extends the corporate debt pricing model in Merton (1974) to incorporate stochastic volatility (SV) in the underlying firm asset value and presents a closed-form solution for the price of corporate debt. Simulation results show that for realistic parameter values, the SV specification for firm asset value greatly increases the resulting credit spread levels. In particular, for debt maturities of less than or equal to five years, the average increase in credit spread levels is 33 basis points (or equivalently, 32.35%) for a typical firm. Therefore, the SV model addresses one major deficiency of the Merton-type models: namely, at short maturities the Merton model is unable to generate credit spreads high enough to be compatible with those observed in the market.

Field of Research: Bond Markets

JEL Codes: G12, G13 and G33

1. Introduction

The corporate debt market is among the largest financial markets. For example, according to a recent estimate of the Bond Market Association, the total amount of U.S. corporate debt outstanding is more than US$ 3 trillion, and the U.S. corporate debt market has surpassed the U.S. Treasury market as the largest segment of the U.S. fixed income market (Ericsson & Reneby 2001). Most corporate debts command a sizeable spread over the yield of corresponding riskless debts (e.g., U.S. Treasury bills, notes etc.). This spread is called the yield spread. The components of this yield spread have long been a major research interest of financial economists. Academics generally agree that one of the main components of yield spreads is a credit spread that compensates for credit risk, i.e., the possible default and credit downgrade of corporate debt. Given the size of corporate debt market, the potential losses from corporate bond defaults are large. Therefore, both academics and practitioners have a keen interest in accurately measuring the credit risk embedded in corporate debts.

This paper contributes to the fast-growing literature on modeling the credit spread of corporate debt. Currently there are two broadly specified approaches to modeling credit risk. The first approach is based on the value of the firm, where "firm" should be considered as a generic term for the issuer of the bond. This approach specifies a default threshold and models the allocation of residual value upon default exogenously. The

* Dr. Xiaofei Li, School of Administrative Studies, Faculty of Liberal Arts and Professional Studies, York University, 4700 Keele Street, Toronto, Ontario, Canada M3J 1P3. Email: xiaofeil@yorku.ca

models used in this approach are called structural models. Merton (1974) first develops structural models, and they are subsequently extended in Longstaff and Schwartz (1995) and others. The second approach considers default as exogenous and assumes that default will occur when an exogenous Poisson process jumps. The models used in this approach are called reduced form models. Much of the recent work on credit risk modeling follows this approach (see, e.g., Madan & Unal 1994 and Duffie & Singleton 1999).[1] Structural models provide very helpful insights into the qualitative aspects of credit risk modeling, whereas reduced form models can not. On the other hand, the structural approach has proven to be quite difficult to use for practical applications such as valuing individual corporate debt securities. In addition, it is often criticized for its inability to generate credit spreads high enough to be compatible with those observed in the market. In contrast, the reduced form approach has the advantage of being able to match the levels of credit spreads prevailing in the market.

This paper presents a structural model of credit risk. It extends Merton's (1974) basic model to allow the volatility of the firm's asset value to be stochastic. In contrast, all existing structural models assume constant volatility of the firm value (see the extensions of the Merton model discussed in Section 2). Academics have ignored the presence of stochastic volatility (SV) in the underlying firm asset value and its potential impact on credit spread levels. However, the constant volatility of firm value assumption is clearly counterfactual: in reality, the volatility of firm asset value—measured as the sum of book value of firm debt and market value of firm equity—changes over time.[2] Accordingly, in the present paper we directly model the volatility of firm value as stochastic using the setup developed by Merton (1974). This exercise is analogous to the approaches in the option pricing literature that modify the Black and Scholes (1973) model by incorporating SV in the underlying stock price to correct for the Black-Scholes model's wide-documented pricing biases.

We demonstrate that for realistic parameter values the addition of SV to the process for firm value can greatly increase credit spread levels in the Merton model, on average by 33 basis points (bps), or equivalently 32.35%, in the base case for debt maturities of less than or equal to five years. This finding is especially encouraging because it is precisely at shorter maturities that the Merton-type models are most vulnerable to the criticism that their resulting credit spread levels are too low to be realistic. Moreover, the framework developed is both flexible and practical in that it can be extended to allow for possible early default prior to debt maturity. It can also be applied to valuing various types of corporate debt securities and credit derivatives. In addition, SV of firm value can be linked more fundamentally to economic factors (e.g., leverage ratios) that drive the firm asset value process. Finally, the valuation framework presented in this paper has many empirical implications for fixed-income markets. The chief of these is that credit spreads for corporate debts are driven by two factors: a firm asset value factor and a stochastic volatility factor. In contrast, the traditional Merton model implies that credit spreads only depend on a firm value factor. After completing the early draft of this paper, we became aware of several other recent articles valuing corporate debts that allow for both default risk and SV of firm asset value. These include Fouque, Sircar, and Solna (2004) and Gatfaoui (2004). This paper distinguishes itself from each of these other studies in that it is the only one that has a closed-form solution for credit risky corporate debt. Closed-form solutions for corporate bonds are very useful, especially from a practical point of view, since they greatly facilitate computations.

The rest of this paper is organized as follows: we review the empirical performance of the Merton model in Section 2; the SV model and a closed-form solution for corporate discount bond value are given in Section 3; Section 4 reports and discusses the simulation results; and finally, Section 5 concludes.

2. Literature Review

Generally speaking, the Merton model has not done well in empirical tests. On the positive side, Sarig and Warga (1989) find that the term structure of credit spreads predicted by the Merton model is consistent with what they have observed in the data. On the negative side, numerous studies of the Merton model document that the model tends to underestimate credit spread levels, particularly at short maturities. Perhaps the most widely cited empirical study of the Merton model to date is that of Jones, Mason, and Rosenfeld (1984). The model considered in that study is the Merton model for a single issue of non-convertible callable coupon bond with a sinking-fund feature. This study investigates a sample of twenty-seven firms that were found to have a simple capital structure (i.e., one class of stock, no convertible bonds, a small number of debt issues, and no preferred stock) in January 1975. The sample consists of monthly data between January 1975 and January 1981. When comparing prices predicted by the model with market prices, Jones, Mason, and Rosenfeld (1984) find that for the entire sample the average pricing error is 4.5 percent and the absolute values of the errors average 8.5 percent. More importantly, they find that for investment-grade bonds the Merton model performs no better than a naive model that assumes no credit risk. However, the Merton model does exhibit some improvement over the naive model when applied to speculative-grade bonds. Overall, these authors conclude that the Merton model overprices corporate debt, or equivalently, underestimates credit spread levels. The finding in Ogden (1987) echoes that of Jones, Mason, and Rosenfeld (1984). He examines a sample of fifty-seven bond offerings and their transaction prices during the years 1973 - 1985. Ogden analyzes spreads, not bond prices. He reports that the average pricing error of the Merton model is minus 104 bps. Thus, the Merton model appears to undervalue credit spreads. Similarly, Franks and Torous (1989), in their empirical study of U.S. firms in reorganization, document that for typical parameter values the Merton model underestimates credit spread levels by 109 bps—however, their finding is not very surprising since they examine firms in financial distress. Similar evidence, for instance, can also be found in Kim, Ramaswamy, and Sundaresan (1993). This study relies on numerical simulations, showing that even with excessive values for the "quasi" leverage ratio and firm value volatility, the maximum credit spread level produced by the Merton model for a ten-year corporate bond with an annual coupon rate of 9 percent is no higher than 120 bps. In contrast, over the 1926-1986 period, the yield spreads on AAA-rated bonds had a range of 15 to 215 bps with an average of 77 bps, while the yield spreads on BAA-rated bonds (also investment-graded) ranged from 51 to 787 bps and had a mean of 198 bps (see Kim, Ramaswamy & Sundaresan 1993).

More recent studies that use better quality bond data than the earlier ones reach the same conclusion. For instance, Eom, Helwege, and Huang (2004) find that the spreads predicted by the Merton model are significantly lower than the observed spreads, particularly for bonds of high rating and shorter maturity, and bonds issued by firms with low volatility. According to the monthly Lehman Brothers Bond Index Data from 1973 to 1993, historically the average yield spreads of ten-year corporate bonds of various credit ratings over U.S. Treasury securities of similar maturities are: Aaa: 63 bps; Aa: 91 bps; A: 123 bps; Baa: 194 bps; Ba: 299 bps; and B: 408 bps. Using numerical simulations, Huang and Huang (2000) report that for the Merton model, the portion of spreads that is attributable to credit risk is: Aaa: 8 bps; Aa: 10 bps; A: 14.3 bps; Baa: 32 bps; Ba: 137.9 bps; and B: 363.3 bps. Since credit risk is assumed to be the only source of yield spread in the Merton model, their simulation results serve as indirect evidence that the Merton model underestimates spread levels, especially for high-rated bonds. However, it is apparent from Huang and Huang's (2000) findings that the Merton model does reasonably well for low-rated bonds.

In this paper we test the following two hypotheses: first, the incorporation of SV in firm value increases the levels of credit spreads generated by the Merton model; second, the inclusion of SV of firm value alters the shape of the credit spread curve in the Merton model, especially at shorter maturities.

3. A Stochastic Volatility Model of Corporate Debt

In this section, we introduce an alternative structural model of credit risk with SV in firm asset value and present a closed-form solution for corporate debt value. Similar to all the existing extensions of the Merton model, we relax one of the basic assumptions made in Merton (1974). In particular, we maintain all the assumptions of Merton (1974) except allowing for SV in firm value. Specifically, we assume that under the risk-neutral probability measure, firm asset value at time t, V_t, follows the diffusion

$$dV_t = rV_t dt + \sqrt{\xi_t} V_t dz_{1t},$$

where z_{1t} is a standard Wiener process. The instantaneous variance of the return on firm value ξ_t is given by a familiar Cox, Ingersoll, and Ross (1985)-type mean-reverting square-root process

$$d\xi_t = \kappa(\theta - \xi_t)dt + \eta\sqrt{\xi_t} dz_{2t},$$

where κ is the mean-reversion speed of the instantaneous variance, θ is the long-run mean level of variance, η is the "volatility of volatility" parameter, and z_{2t} is another standard Wiener process which has an instantaneous correlation coefficient ρ with z_{1t}. More sophisticated SV models for firm asset value than the model above can be developed (see, e.g., Fouque, Sircar & Solna 2004 and Gatfaoui 2004). However, in these more general models there are no closed-form solutions for credit risky debt prices. Since our main objective is to provide a tractable valuation model for credit risky securities and to examine the impact of SV of firm value on credit spread levels, we adopt the above simpler specification for firm value and its variance. The present model can be regarded as a two-factor model of corporate debt, where the two factors are firm asset value and its instantaneous SV, respectively. In contrast, the Merton model is a one-factor model where the only factor underpinning the credit risk is firm value. Our model allows for a closed-form solution to default risky corporate bond prices. The exact pricing formula is available upon request.

4. The Findings

Table 1 contains the base case parameter values used in the simulations. The current annual variance of firm value is set at 0.1, which corresponds to an annualized volatility (standard variation) of 0.316. This is consistent with Jones, Mason, and Rosenfeld (1984), who estimate that the individual firms in their sample have an annualized volatility of firm value ranging from 0.052 to 0.663 (see their Table 1). For the base case the current variance is set at its long-run mean level, because of the assumed mean-reversion in firm value variance levels. An annualized riskless interest rate of 6 percent is used, close to that in Longstaff and Schwartz (1995) and Zhou (2001). Finally, a negative correlation coefficient (ρ = -0.5) is chosen between the value of the firm and the SV of firm value. That is, we assume that firm asset value and its volatility are negatively correlated. It must be noted that the validity of this assumption and the relationship between firm value movement and its volatility are an open empirical question. This paucity of evidence is partially due to the difficulty in measuring firm value accurately, since it is unobservable and in reality firms have very complex capital structures. The usual practice of measuring firm value as the sum of a firm's book-value of debt and market-value of equity is only an approximation. On the other hand, the "leverage effect" in stock price volatility has been widely documented in the empirical literature (see among others, Black 1976 and Christie 1982).

Table 1: Base Case Parameter Values

Parameter	Value
mean reversion speed	0.5
long-run mean of variance	0.1
current variance	0.1
correlation between z_{1t} and z_{2t}	-0.5
volatility of volatility	0.225
annual riskless interest rate	0.06
current firm asset value	100

Under the risk-neutral probability measure, the firm asset value and its variance are assumed to follow the joint diffusion processes below:

$$dV_t = rV_t dt + \sqrt{\xi_t} V_t dz_{1t},$$
$$d\xi_t = \kappa(\theta - \xi_t)dt + \eta\sqrt{\xi_t} dz_{2t}, \textbf{ where } dz_{1t}dz_{2t} = \rho dt.$$

Table 2 presents, for the base case parameters, the yield spreads generated by the SV model, those produced by the Merton model, and the difference between the two when *d*, the "quasi" debt-firm value ratio (or leverage ratio), is set at 0.2 or 0.5. The values of *d* are identical to those used in Table 1 in Merton (1974, p. 457). In addition, we consider six debt maturities: 0.5, 1, 2, 5, 10, and 15 years. Figure 1 gives a graphic illustration of the difference in yield spreads. Several findings are noteworthy. First, it is evident from both the table and the figure that the maximal increase in yield spread levels resulted from incorporating SV of firm value into the Merton model occurs at maturities of less than or equal to five years. At these short maturities, the SV specification for firm asset value increases the resulting credit spread levels by an average of 10 bps when *d* = 0.2 and 33 bps for *d* = 0.5. The significance of these increases is best illustrated in relative terms. For debts with maturities less than or equal to five years, the average yield spread generated by the Merton model is only 3.4 bps when *d* = 0.2 and is 102 bps when *d* = 0.5. Therefore, by incorporating SV of firm value, the yield spread levels increase compared to the Merton model on average by 294% for *d* = 0.2 and 32.35% for *d* = 0.5! In addition, when *d* = 0.5 the yield spread curve peaks at around 2.5 years. These results are particularly encouraging because, as suggested by the empirical evidence in Section 2, it is precisely at short maturities that the Merton-type models have greatest difficulty in matching the observed credit spread levels. Our first hypothesis, namely, the incorporation of SV in firm value increases the levels of credit spreads generated by the Merton model, is therefore supported by the simulation results. The results also show that the SV specification for firm value can substantially alter the shape of the credit spread curve predicted by the Merton model, especially at shorter maturities. Thus, our second hypothesis, i.e., the inclusion of SV of firm value alters the shape of the credit spread curve in the Merton model, particularly for shorter maturities, is also supported by the simulation results.

Second, by incorporating SV of firm value into the Merton model, we can increase the resulting credit spread levels by a sizeable amount across all maturities: on average 16 bps when *d* = 0.2 and 12 bps for *d* = 0.5; or in relative terms, on average 48.48% for *d* = 0.2 and 6.9% for *d* = 0.5. For instance, when *d* is set at 0.2, for a debt with maturity of 5 years, the Merton model generates a credit spread of merely 12 bps, whereas the SV model generates 35 bps, almost triple that of Merton. Likewise, at 1 year of maturity, for a firm with *d* of 0.5, the SV model's credit spread (55 bps) is far more than twice as large as in the Merton model (22 bps). Since *d* is an upward biased estimate of the actual leverage ratios, when we set *d* equal to 0.2 and 0.5, we are essentially examining firms with low and medium leverage ratios. Roughly speaking, bonds of these firms are of investment-grade: BBB or higher according to Standard & Poor's, or Baa or higher according to Moody's. In fact, it is reasonable to say that firms with *d* of 0.2 have bonds of highest ratings, for instance, AAA according to Standard & Poor's or Aaa according to

Moody's, because the AAA-rated firms had a historical average leverage ratio of 13.08% (Standard & Poor's 1999). Since it is clear from the empirical evidence in Section 2 that the Merton model tends to underestimate spread levels for high-rated bonds, our study shows that the specification for SV in firm value can help significantly increase Merton's yield spreads for firms with high-rated bonds.[3]

Table 2: Base Case Results

Panel A: *d* = 0.2	Debt maturity (in years)	SV's YS (in basis points)	Merton's YS (in basis points)	YS difference (in basis points)
	0.5	-4	0	-4
	1	-2	0	-2
	2	4	0	4
	5	35	12	23
	10	68	47	21
	15	86	75	11
Panel B: *d* = 0.5	**Debt maturity (in years)**	**SV's YS (in basis points)**	**Merton's YS (in basis points)**	**YS difference (in basis points)**
	0.5	9	2	7
	1	55	22	33
	2	129	82	47
	5	196	174	22
	10	211	211	0
	15	211	219	-8

The above two panels contain the simulation results obtained using the base case parameter values given in Table 1, when *d*, the “quasi” debt-firm value ratio, is set at 0.2 and 0.5, respectively. The SV’s YS is the corporate debt yield spread generated by the SV model. Similarly, the Merton’s YS is the corporate bond yield spread generated by the Merton model. The YS difference is the yield spread of the SV model minus that of the Merton model. All the numbers have been rounded to the nearest decimal point.

Third, as time to maturity lengthens, the effect of SV in firm value diminishes and the yield spreads of the SV model converge with those of the Merton model. This is as expected since SV is mean reverting, and thus will be pulled back to its long-run mean level, θ, as time to maturity increases. It then follows that in the long run, the impact of SV of firm value on credit spread levels will diminish.[4] Finally, as panel A of Table 2 indicates, for firms with very low leverage ratios (*d* = 0.2 in the table), and for very short maturities (maturities of 0.5 and 1 year in the table), the SV model may generate credit spreads lower than those of the Merton model, which are essentially zero. The explanation for this finding is the following. At very short maturities, a marginally levered firm can be thought of as being well above its default boundary and the likelihood of it defaulting during the time remaining to maturity is very low, because of the assumed diffusion process for the firm asset value.[5] This explains the almost zero credit spreads generated by the Merton model, since in the Merton model default risk is assumed to be the only source of credit spreads. On the other hand, the effects of SV on credit spreads take time to "materialize," due to the assumed stochastic process for the volatility. It follows that the SV model may not be able to improve on the Merton's credit spread levels in this case.[6,7] In contrast, for firms with medium leverage ratios (*d* = 0.5 in panel B of Table 2), the SV specification for firm asset value largely increases the Merton's credit spread levels at every debt maturity except the longest two (10 and 15 years in the table).

To verify whether the parameter values used in our simulations are realistic, we calculate in Table 3 the equity price volatility in the Merton model that is implied by the chosen parameter values. Following Jones, Mason, and Rosenfeld (1984), we compute this volatility as follows

$$\sigma_E = \sqrt{\xi}\,\frac{V\dfrac{\partial E}{\partial V}}{E}$$

where σ_E and $\sqrt{\xi}$ refer to the volatilities of equity price and firm value, respectively. The majority of equity price volatilities obtained in Table 3 are in the range of 38% to 60%, slightly higher than the value for the annualized actual equity volatility of 20% to 40% documented in Hull (2000). Therefore, we conclude that our parameter values are reasonable.

Table 3: Implied Merton Volatilities

Debt maturity (in years)	Implied Merton volatility $d = 0.2$ (in percentage points)	Implied Merton volatility $d = 0.5$ (in percentage points)
0.5	40	63
1	40	63
2	40	60
5	39	53
10	38	47
15	37	44

The implied Merton volatility is the equity price volatility in the Merton model that is implied by the parameter values chosen for the base case. All the numbers have been rounded to the nearest decimal point.

Figure 2 reports the risk-neutral conditional cumulative default probabilities calculated for both the Merton and SV models. Three findings are noteworthy. First, the SV model generates a higher default probability than the Merton model for debts with maturities less than or equal to three years; however, the difference between the two models' default probabilities is small (on average about 2% across maturities). (In the results not reported, when d is set at 0.2, the cumulative default probability in the SV model is higher than that in the Merton model at almost every maturity, with the largest probability difference of around 2.6% occurring at a maturity of about 5 years.) On the other hand, we see from panel B in Table 2 that the SV model produces significantly higher yield spreads than the Merton model for debts with maturities less than or equal to five years (on average 33 bps). Because this is an important result, we now offer the intuition for it.

In both the SV and Merton models, credit risk affects yield spread in two forms: through the default probability and through the recovery rate of debt when default occurs. Obviously, when calculating credit spreads in reality, the recovery rate is as important as the default probability: a higher (resp. lower) default probability, coupled with a lower (resp. higher) recovery rate, naturally leads to a lower (resp. higher) debt price (and a higher (resp. lower) yield spread), all else being equal; on the other hand, a higher default probability, but together with a sufficiently higher recovery rate, may actually result in a higher debt price (and a lower yield spread) when compared to a case where both the default probability and the recovery rate are sufficiently lower, all else being the same. In both the Merton and SV models, the recovery rate upon default depends on the firm's remaining asset value at that time. Since both models preclude early default prior to debt maturity (i.e., default can only occur when debt matures), the firm's asset value upon default is stochastic. It then follows that in both models the recovery rate of debt is also stochastic. As mentioned in Section 3, the SV model has one more factor underlying credit risk—the SV of firm asset value—when compared to the Merton model, which has only one factor—firm value. Therefore, it is reasonable to believe that the SV model can add more variations to credit risk, resulting in not only a more variable default probability, but also a more volatile recovery rate compared to the Merton model.

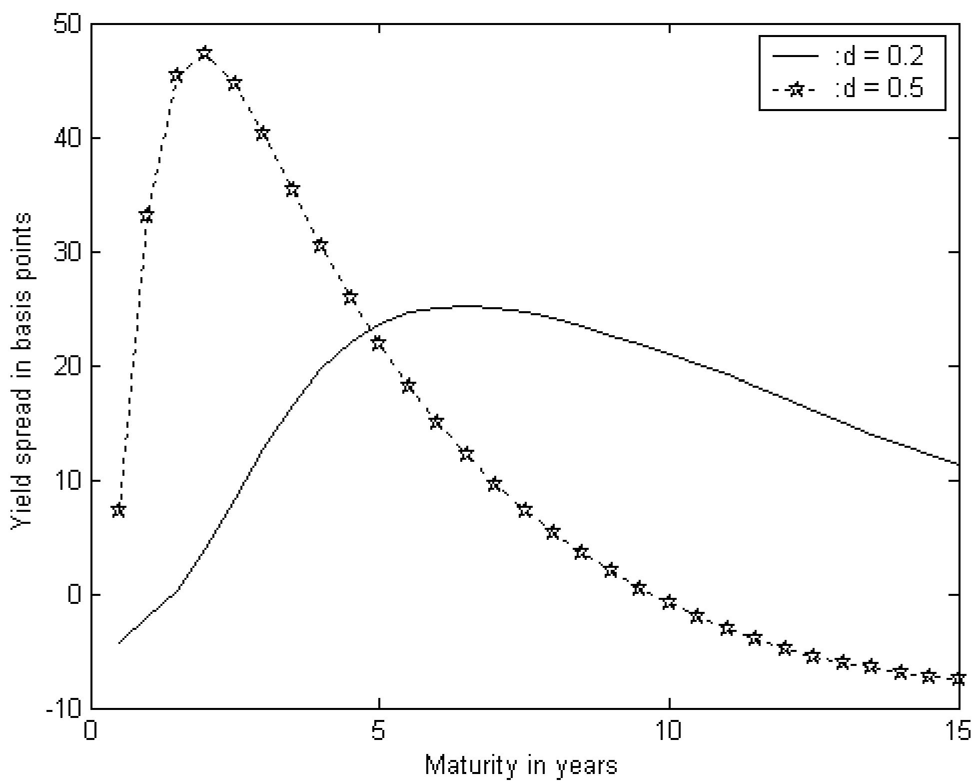
To verify the above intuition, we run some Monte Carlo simulations (with 1,000 replications) to compute the mean recovery rates in the SV and Merton models.[8] The results are reproduced graphically in Figure 3. Figure 4 contains the results obtained for debts with maturities less than or equal to fives years. Consistent with our intuition, both Figure 3 and Figure 4 show that the SV model generates a lower mean recovery rate than that in the Merton model for maturities less than or equal to five years; for maturities over five years, these two models produce almost indistinguishable mean recovery rates. In a parallel Monte Carlo simulation exercise, we find that the volatility of the recovery rate in the SV model is generally higher than that in the Merton model, again consistent with our intuition.[9] Therefore, a small difference in default probabilities (as shown in Figure 2) can lead to a much larger difference in credit spread levels (in panel B of Table 2). A numerical example helps to further illustrate this point. In the base case, a bond with a maturity of 2 years has a risk-neutral conditional cumulative default probability of 10.57% in the SV model, while the corresponding probability in the Merton model is 9.24%. So the difference in probabilities is 1.33%. Monte Carlo simulations show that this bond has a recovery rate of 39.6% of par in the SV model and a recovery rate of 41.6% of par in the Merton model.[10] The difference in recovery rates is therefore -2%. Under the risk-neutral measure, we calculate the bond price as the expected payoffs of the bond (in the event of default and in the event of no default) discounted at the risk free interest rate for the time to maturity. The corresponding yield spreads in the two models can then be readily computed. The difference in yield spreads is found to be 53 bps, only slightly higher than that reported in panel B of Table 2, which is 47 bps.

Second, although the magnitudes of the default probabilities reported in Figure 2 appear to be large, they are indeed consistent with those found in Jarrow, Lando, and Turnbull (1997). Our probability estimates also correspond approximately to the Moody's historical cumulative default rates (1970 – 1993) for Ba- and B-rated firms reported in Huang and Huang (2000). This result provides additional evidence that the parameter values chosen for this paper are reasonable. Also notice that the default probabilities presented in Figure 2 are the risk-neutral ones, which are found to be much higher than the actual default probabilities (see Duffee 1999). Finally, notice that in Figure 2 over longer period of time (specifically, at maturities of more than five years), the Merton model actually produces a higher default probability than the SV model, though the difference is again not large. On the other hand, Figure 3 provides weak evidence that at maturities longer than eight years, the SV model generates a higher mean recovery rate than the Merton model. Together, these two facts partially explain the observations in Table 2 and Figure 1 that the impact of SV on credit spreads diminishes at long maturities.

5. Conclusion

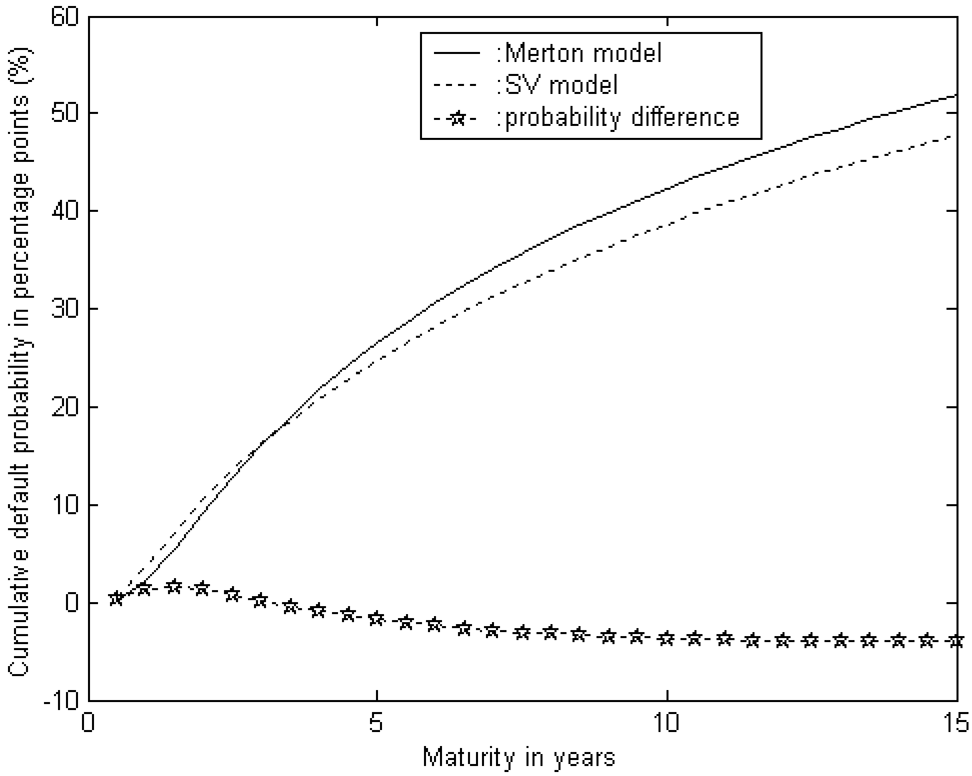
This paper extends the seminal work of Merton (1974) by developing a structural model of corporate debt that incorporates the stochastic volatility of firm asset value. We present a closed-form solution for the resulting corporate debt price. Computational experiments show that for realistic parameter values incorporating SV of firm value can substantially increase the generated credit spread levels in both absolute and relative terms. The largest increase in spreads occurs for debts with maturities less than or equal to five years, even for firms with a low or medium leverage ratio: for a typical firm an increase on average of 33 bps, or equivalently, 32.35% of the original credit spread levels in the Merton model. Both of our hypotheses are supported by the simulation results. This finding is encouraging since it is well known that contrary to observed credit spread levels, the credit spreads predicted by the Merton model are not much higher than zero for short-maturity debts, especially for high-grade ones. Finally, volatility plays an important role in, among other things, asset valuation, portfolio diversification, and risk management. Since our SV model can better fit the empirical behavior of credit spreads, it may be a useful tool in all these applications.

Figure 1: Base Case Yield Spread Differences



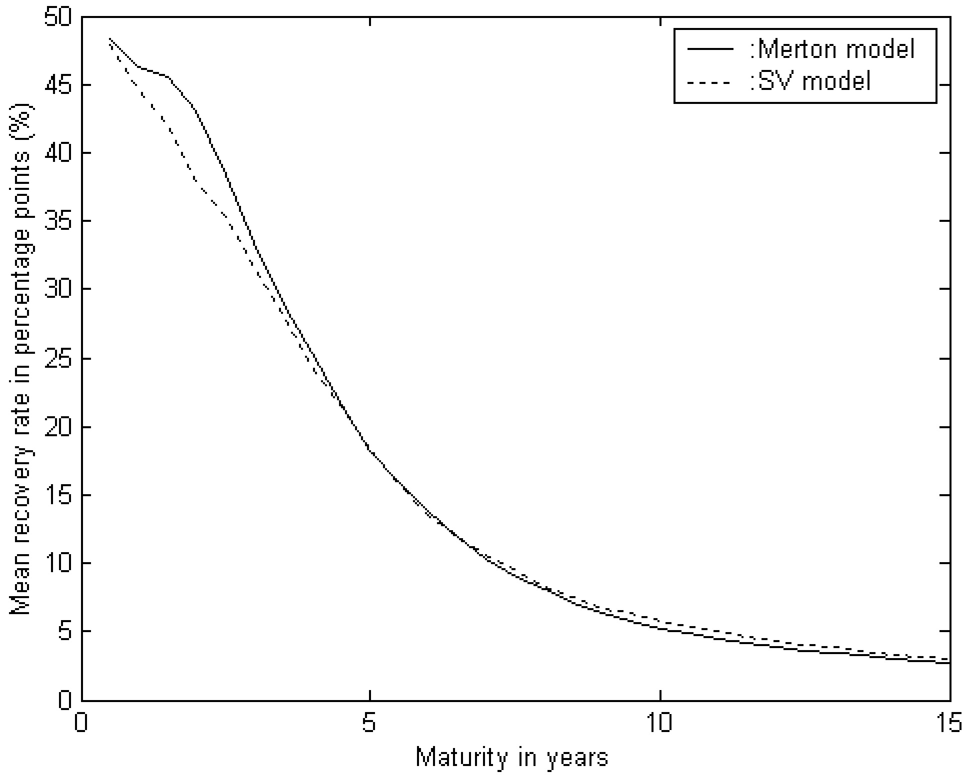
Yield spread of the SV model minus that of the Merton model when *d*, the "quasi" debt-firm value ratio, is set at 0.2 and 0.5, respectively. Base case parameters: $\kappa = 0.5$, $\xi_t = \theta = 0.1$, $\eta = 0.225$, $\rho = -0.5$, $r = 0.06$, and $V = 100$.

Figure 2: Cumulative Default Probabilities



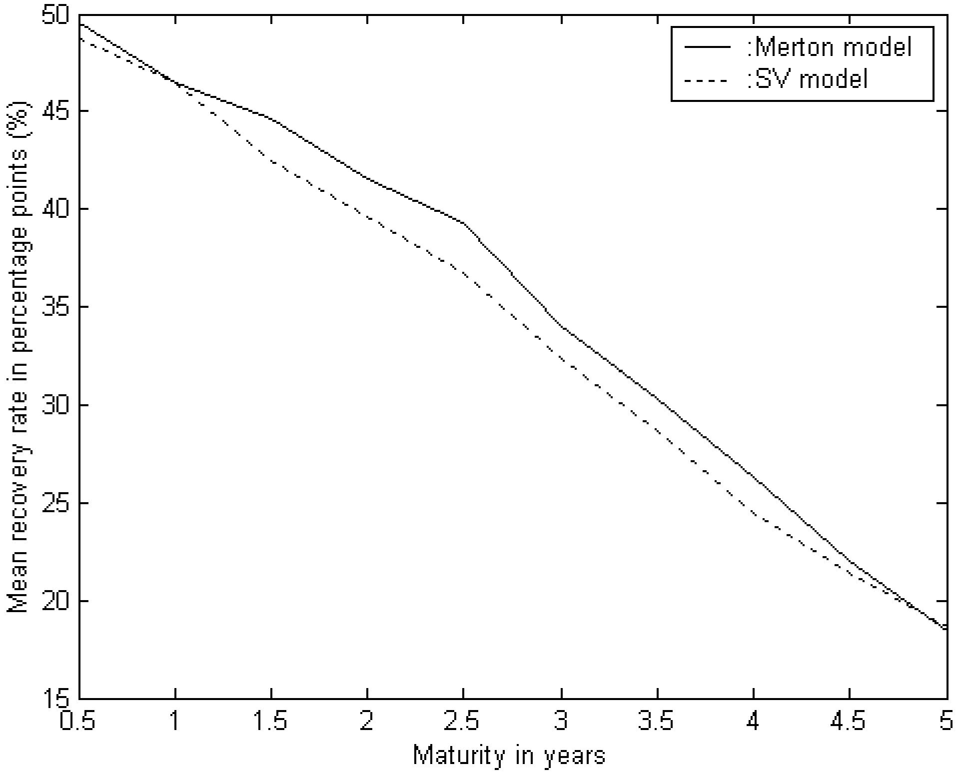
Risk-neutral conditional cumulative default probabilities of the SV model and the Merton model. The probability difference is the probability of the SV model minus that of the Merton model. Base case parameters: $\kappa = 0.5$, $\xi_t = \theta = 0.1$, $\eta = 0.225$, $\rho = -0.5$, $d = 0.5$, $r = 0.06$, and $V = 100$.

Figure 3: Mean Recovery Rates



Mean recovery rates in the SV and Merton models, which are calculated using Monte Carlo simulations (with 1,000 replications). Base case parameters: $\kappa = 0.5$, $\xi_t = \theta = 0.1$, $\eta = 0.225$, $\rho = -0.5$, $r = 0.06$, $d = 0.5$, and $V = 100$.

Figure 4: Mean Recovery Rates for Maturities less than or Equal to Five Years



Mean recovery rates in the SV and Merton models for debts with maturities less than or equal to five years. The recovery rates are calculated using Monte Carlo simulations (with 1,000 replications). Parameter values are the same as those used in Figure 3.

Endnotes

1. Sundaresan (2000) reviews the existing structural and reduced form models.
2. It is worth pointing out that since firm value is unobservable, it is difficult, if not impossible, to measure the volatility of firm value precisely in practice. However, an indication of the time-varying volatility of firm value is the well-established fact that the volatility of market equity price is stochastic (Black 1976).
3. In this paper we focus on investment-grade bonds. The properties of credit spreads for speculative-grade bonds are studied in, e.g., Sarig and Warga (1989), Fons (1994), and Helwege and Turner (1999). In general, the evidence on the credit spreads for speculative-grade bonds is inconclusive.
4. The Merton model generates counterfactual near-zero credit spreads at sufficiently long maturities (not shown in the table). One possible explanation for this problem concerns the fact that in the Merton model the firm leverage ratios are non-stationary (Collin-Dufresne & Goldstein 2001). It has to be said that the SV model is not free from this problem either. Developing a structural model of corporate debt in which the firm's leverage ratio is assumed to be stationary, while at the same time allowing firm value volatility to be stochastic, is a fruitful path for future research.
5. The Merton-type models (including the SV model) all assume that firm value follows a diffusion process. Under a diffusion process, default can only be triggered by a gradual decline in firm asset value over a longer time horizon. Therefore, in these models default can only occur expectedly.
6. This result also complements that in Hull (2000), who finds that for equity options of maturities of less than 1 year, the pricing impact of SV is not large, especially in absolute terms.
7. In panel A of Table 2, when d = 0.2, for maturities of 0.5 and 1 year the SV model generates negative credit spreads (though they are economically insignificant). This result is counter-intuitive since it implies

that default risky debts are more valuable than the corresponding default-free bonds. We believe that this anomaly is caused by the slight inaccuracy of the numerical algorithm used, which is inevitable in this type of analysis. It must be noted that the main conclusions of this paper still hold, despite this result. In addition, when d = 0.5 the SV model leads to positive credit spreads across maturities.

8. The results obtained after 5,000 replications of Monte Carlo simulations (not reported) are virtually indistinguishable. These results are available upon request.

9. The results of this parallel Monte Carlo exercise are available upon request.

10. According to Moody's estimate, senior unsecured debts have an average recovery rate of 44% of par in the event of default.

References

Black, F 1976, 'Studies of stock price volatility changes', *Proceedings of the 1976 meetings of the American Statistical Association, Business and Economic Statistics Section,* pp. 177-181.

Black, F & Scholes, M 1973, 'The pricing of options and corporate liabilities', *Journal of Political Economy*, vol. 81, no. 3, pp. 637-654.

Christie, AA 1982, 'The stochastic behavior of common stock variances: value, leverage and interest rate effects', *Journal of Financial Economics*, vol. 10, no. 4, pp. 407-432.

Collin-Dufresne, P & Goldstein, RS 2001, 'Do credit spreads reflect stationary leverage ratios?', *Journal of Finance*, vol. 56, no. 5, pp. 1929-1957.

Cooper, I & Mello, A 1991, 'The default risk of swaps', *Journal of Finance*, vol. 46, no. 2, pp. 597-620.

Cox, JC, Ingersoll, JE Jr. & Ross, SA 1985, 'A theory of the term structure of interest rates', *Econometrica*, vol. 53, no. 2, pp. 385-407.

Duffee, GR 1999, 'Estimating the price of default risk', *Review of Financial Studies*, vol. 12, no. 1, pp. 197-226.

Duffie, D & Singleton, KJ 1999, 'Modeling term structure of defaultable bonds', *Review of Financial Studies*, vol. 12, no. 4, pp. 687-720.

Eom, YH, Helwege, J & Huang, JZ 2004, 'Structural models of corporate bond pricing: an empirical analysis', *Review of Financial Studies,* vol. 17, no.2, pp. 499-544.

Ericsson, J & Reneby, J 2001, 'The valuation of corporate liabilities: theory and tests', Working Paper, McGill University and Stockholm School of Economics, Montreal and Stockholm.

Fons, JS 1994, 'Using default rates to model the term structure of credit risk', *Financial Analysts Journal*, vol. 50, no. 5, pp. 25-32.

Fouque, JP, Sircar, R & Solna, K 2004, 'Stochastic volatility effects on defaultable bonds', Working Paper, North Carolina State University, Princeton University, and University of California (Irvine), Raleigh, Princeton, and Irvine.

Franks, JR & Torous, WN 1989, 'An empirical investigation of U.S. firms in reorganization', *Journal of Finance*, vol. 44, no. 3, pp. 747-769.

Gatfaoui, H 2004, 'Idiosyncratic risk, systematic risk and stochastic volatility: an implementation of Merton's credit risk valuation', Working Paper, University Paris I - Pantheon-Sorbonne, Paris.

Helwege, J & Turner, CM 1999, 'The slope of the credit yield curve for speculative-grade issues', *Journal of Finance*, vol. 54, no. 5, pp. 1869-1884.

Huang, M & Huang, JZ 2000, 'How much of the corporate-treasury yield spread is due to credit risk?: results from a new calibration approach', Working Paper, Stanford University and Pennsylvania State University, Stanford and University Park, Pennsylvania.

Hull, JC 2000, *Options, Futures, and Other Derivatives,* 4th edn, Prentice Hall, Upper Saddle River, N.J.

Jarrow, RA, Lando, D & Turnbull, SM 1997, 'A Markov chain model for the term structure of credit risk spreads', *Review of Financial Studies*, vol. 10, no. 2, pp. 481-523.

Jones, EP, Mason, SP & Rosenfeld, E 1984, 'Contingent claims analysis of corporate capital structure: an empirical investigation', *Journal of Finance*, vol. 39, no. 3, pp. 611-625.

Kim, J, Ramaswamy, K & Sundaresan, S 1993, 'Does default risk in coupons affect the valuation of corporate bonds?: a contingent claim model', *Financial Management*, vol. 22, no. 3, pp. 117-131.

Longstaff, FA & Schwartz, ES 1995, 'A simple approach to valuing risky fixed and floating rate debt', *Journal of Finance*, vol. 50, no. 3, pp. 789-819.

Madan, D and Unal, H 1994, 'Pricing the risks of default', Working Paper 94-16-B, Wharton Financial Institutions Center, Philadelphia.

Merton, RC 1974, 'On the pricing of corporate debt: the risk structure of interest rates', *Journal of Finance*, vol. 29, no. 2, pp. 449-470.

Ogden, JP 1987, 'Determinants of the ratings and yields on corporate bonds: tests of the contingent claims model', *Journal of Financial Research*, vol. 10, no. 3, pp. 329-339.

Sarig, O & Warga, A 1989, 'Some empirical estimates of the risk structure of interest rates', *Journal of Finance*, vol. 44, no. 5, pp. 1351-1360.

Standard & Poor's 1999, *Corporate Ratings Criteria*.

Sundaresan, SM 2000, 'Continuous-time methods in finance: a review and an assessment', *Journal of Finance*, vol. 55, no. 4, pp. 1569-1622.

Zhou, CS 2001, 'The term structure of credit spreads with jump risk', *Journal of Banking and Finance*, vol. 25, no. 11, pp. 2015-2040.